1) On a limit problem

We begin with the formulation of a problem, which is important both for probability theory and statistical physics. The multiple Wiener-Itô integral proved to be a very useful tool at the investigation of this problem.

Let \( \xi_n', n \in \mathbb{Z}_v \), where \( \mathbb{Z}_v \) denotes the \( v \)-dimensional integer lattice, be a discrete (strictly) stationary random field. We recall that a set of random variables \( \xi_n', n \in \mathbb{Z}_v \), is called a (discrete) random field. It is called (strictly) stationary if \( (\xi_{n_1}', \ldots, \xi_{n_k}') \overset{\Delta}{=} (\xi_{n_1}'+m', \ldots, \xi_{n_k}'+m') \) for all \( k=1,2,\ldots \) and \( n_1', \ldots, n_k', m \in \mathbb{Z}_v \), where \( \overset{\Delta}{=} \) denotes equality in distribution. For all \( N=1,2,\ldots \) we define the new fields

\[
Z_n^N = A^{-1}_N \sum_{j \in B_n^N} \xi_j, \quad N=1,2,\ldots \quad n \in \mathbb{Z}_v
\]

where

\[
B_n^N = \{ j | j \in \mathbb{Z}_v, n(i)N \leq j(i) < (n(i)+1)N, i=1,2,\ldots,v \}
\]

(the superscript \( i \) denotes the \( i \)-th coordinate of a vector), and \( A_N \) is an appropriate norming constant.

We are interested in the following questions:

When do the finite dimensional distributions of the fields \( Z_n^N \) converge to the finite dimensional distributions of a field \( Z_n^\infty \)? Which fields \( Z_n^\infty \) can appear as limits?
During the investigation of the above questions one also has to solve the following problem: Which fields $\xi_n$ satisfy the relation

\[(\xi_{n_1}, \ldots, \xi_{n_k}) \overset{\Delta}{=} (z_{n_1}^N, \ldots, z_{n_k}^N)\]

for all $N=1,2,\ldots$ and $n_1, \ldots, n_k \in \mathbb{Z}_+$. If the field $\xi_n$ satisfies relation (1.2) with $A_N = N^\alpha$ then $\xi_n$ (or its distribution) is called a self-similar field with self-similarity parameter $\alpha$. We are interested in the case $A_N = N^\alpha$ because, under some slight restrictions, for fields $\xi_n$ satisfying (1.2) $A_N$ must be chosen in this way.

A central problem both in statistical physics and in probability theory is the description of self-similar fields. We are interested in self-similar fields whose random variables have a finite second moment. This excludes the fields consisting of i.i.d. random variables with a non-Gaussian stable law.

The Gaussian self-similar fields and their Gaussian range of attraction are fairly well known. Much less is known about the non-Gaussian case. The problem is hard, because the transformations of measures over $\mathbb{Z}_+$ induced by formula (1.1) have a very complicated structure. We shall define the so-called subordinated fields below. (More precisely the fields subordinated to a stationary Gaussian field). In case of subordinated fields the Wiener-Itô integral is a very useful tool for investigating the