In this chapter are the most important theorems of the preceding chapters translated into the language of preschemes. Since the proofs, of course, reduce immediately to the affine case, no further proof is required for the versions presented in this chapter. We begin with the preliminaries of defining simple, unramified and étale morphisms of preschemes. It is assumed here that the reader is already well-acquainted with the language and fundamentals of preschemes.

10.1. Definition. Let $k$ be a field, $X$ be a prescheme over $k$ and $x \in X$. We say that $X$ is simple over $k$ at $x$ if and only if there exists an affine open neighborhood $U$ of $x$ in $X$ such that the ring of $U$ is simple over $k$ at the prime ideal corresponding to $x$. We say that $X$ is simple over $k$ if and only if $X$ is simple over $k$ at each point $x$ of $X$.

10.2. Definition. Let $X$ and $Y$ be preschemes, $f : X \rightarrow Y$ be a morphism of preschemes, $x \in X$ and put $y = f(x)$. We say that $f$ is simple at $x$ or $X$ is simple over $Y$ at $x$ if and only if $f$ is locally of finite presentation in an open neighborhood of $x$ in $X$, $f$ is flat at $x$ and $X \times \text{Spec} \ (k(y))$ is simple over $k(y)$ at $x$. We say that $f$ is simple or $X$ is simple over $Y$ if and only if $f$ is simple at each point $x$ of $X$. 
10.3. Definition. Let \( X, Y, f \& x \) be as in Def. 10.2. We say that \( f \) is **unramified at \( x \)** or \( X \) is **unramified over \( Y \) at \( x \)** if and only if \( f \) is locally of finite presentation in an open neighborhood of \( x \) in \( X \) and 
\[
\Gamma^1_Y(X)_x = 0.
\]
(See Remark 10.3.1 below for the definition of the sheaf of \( \mathcal{O}_X \)-modules \( \Gamma^1_Y(X) \).) We say that \( f \) is **étale at \( x \)** or \( X \) is **étale over \( Y \) at \( x \)** if and only if \( f \) is flat and unramified at \( x \). We say that \( f \) is **unramified** or \( X \) is **unramified over \( Y \)** (resp., \( f \) is **étale** or \( X \) is **étale over \( Y \)**) if and only if \( f \) is unramified (resp., étale) at all points \( x \) of \( X \).

10.3.1. Remark. Let \( X, Y \) and \( f \) be as in Def. 10.2. The diagonal morphism \( X \to X \times_Y X \) being an immersion is a closed immersion \( X \to V \) for some open subset \( V \) of \( X \times_X X \). Let \( I \) be the sheaf of ideals of \( \mathcal{O}_X \) defining the closed subprescheme of \( X \) corresponding to the diagonal immersion into \( V \). The sheaf of \( \mathcal{O}_X \)-modules \( I/I^2 \) is denoted \( \Gamma^1_Y(X) \) and is called the **sheaf of Kahler 1-differentials** of the \( Y \)-prescheme \( X \).

Note that if \( X = \text{Spec}(B) \) and \( Y = \text{Spec}(A) \) are affine then by construction and Def. 2.1 we conclude that \( \Gamma^1_Y(X) = \Gamma^1_A(B) \). Moreover, in the general case given \( x \in X \), putting \( y = f(x) \), choosing an affine open neighborhood \( W \) of \( y \) in \( Y \) with ring \( A \) and an affine open neighborhood \( U \) of \( x \) in \( X \) with ring \( B \) and letting \( Q \in \text{Spec}(B) \) be the prime ideal corresponding to \( x \) we have 
\[
\Gamma^1_Y(X)_x = \Gamma^1_A(B)_Q.
\]
Hence if \( X \) and \( Y \) are both affine then Def. 10.1, Def. 10.2 and Def. 10.3 agree with the corresponding definitions in earlier chapters.