0. Introduction. Let \((B, \mathcal{B}(B), \mu)\) be a real separable Banach space with a mean-zero Gaussian measure \(\mu\) defined on the Borel \(\sigma\)-algebra \(\mathcal{B}(B)\). It is well known that for these spaces the measure \(\mu\) can be obtained as a \(\sigma\)-extension of the canonical cylindrical Gaussian measure \(\mu_H\) concentrated on a separable Hilbert space \(H_\mu\) such that

\[
B^* \subseteq H^*_\mu = H_\mu \\subsetneq B.
\]

The embedding of \(H_\mu\) into \(B\) is a continuous linear injection, and each continuous linear functional on \(B\) restricted to the space \(H_\mu\) is continuous on \(H_\mu\). Furthermore the measure \(\mu\) is concentrated on the closure of \(H_\mu\) in the norm of \(B\).

The above problem for separable Banach spaces was considered by many authors: L. Gross [8], J. Kuelbs [12], H. Sato [21], B. Rajput [18]. In 1976 the above results were generalized to locally convex Hausdorff real vector spaces by Ch. Borell [2].

It is worth pointing out that the techniques used for the proofs of these results heavily depend on various properties of locally convex vector spaces. And of course the dual space of continuous linear functional plays here an important role. However, there are metric linear spaces, natural from the point of view of probability theory, having no non-trivial continuous linear functionals. The best known examples are the space \(L^0\) of all measurable functions defined on \([0,1]\), with Lebesgue measure \(m\), endowed with the topology of convergence in measure; or \(L^p[0,1]\), \(0 < p < 1\).

The main theme of this presentation is to give the above description of Gaussian measures on some function spaces which are not necessarily Banach spaces, like Orlicz spaces \(L_\phi\) for different \(\phi\).
Let us recall that for every Gaussian random element \( X \) with values in \( C[0,1] \) (separable Banach space in the sup norm) there corresponds a Gaussian stochastic process with sample paths in this space. This correspondence is easy to establish by taking the evaluation \( x \mapsto x(t) \). And conversely Gaussian stochastic processes with sample paths in \( C[0,1] \) can be considered as random elements with values in \( C[0,1] \) (endowed with its Borel \( \sigma \)-field). It turns out that the correspondence between Gaussian processes with sample paths in a linear function space and Gaussian random elements with values in this space is true for a wide class of spaces. It was proved by Rajput for \( L^p; \ p \geq 1 \) in 1972 [17] and in 1976 generalized to Orlicz spaces \( L_\phi \) by Byczkowski [3].

The problem of the correspondence between measures on \( L_\phi \) and measurable stochastic processes with sample paths in \( L_\phi \) is closely related to the problem of the existence of sufficiently many quasi-additive measurable functionals (q.m.f.) on \( L_\phi \) (Definition 1.1). In 1978 Byczkowski [6] proved that for any separable Orlicz space \( L_\phi \) and any non-degenerate probability measure \( \mu \) defined on \( (L_\phi, B(L_\phi)) \) there exist sufficiently many q.m.f., i.e., that they generate \( \sigma \)-algebra \( B(L_\phi) \) (mod \( \mu \)). In a space without sufficiently many continuous linear functionals these q.m.f. provide some analogue of the dual space of a vector space-with-measure \( (L_\phi, B(L_\phi), \mu) \). This substitute will be denoted by \( (L_\phi, B(L_\phi), \mu) \). It turns out that we can define a Gaussian measure on \( L_\phi \) in terms of q.m.f..

In the case when \( L_\phi \) is an Orlicz space of real-valued, measurable functions defined on the real line with Lebesgue measure we can show that each symmetric, non-degenerate, Gaussian measure is the \( \sigma \)-extension of the canonical cylindrical Gaussian measure \( \mu_H \) concentrated on a separable Hilbert space \( H_\mu \) such that the canonical injection of \( H_\mu \) into \( L_\phi \) is continuous. The proof of these results is based on the correspondence between Gaussian measures and Gaussian stochastic processes with sample paths in \( L_\phi \) and on the construction of the canonical measurable stochastic process in the sense of K. Itô (1968) [9], [10].

In a standard construction, the regularized version of a stochastic process equivalent to a given one usually is obtained by the modification of random variables. The novelty in Itô's construction consists of the regularization of the sample paths of the given process.

In spaces \( L_\phi \), the canonical measurable stochastic processes play an important role. The elements of \( L_\phi \) are classes of \( m \)