1. Preliminaries. Until 1974 it was thought by many that a separable Banach space that contains no copy of $l_1$ must have a separable dual. In that year, James [10] put matters straight with one of his celebrated spaces, the James Tree space, thus demonstrating that the class of separable spaces with separable duals and the class of separable spaces containing no copy of $l_1$ are not identical classes.

This fact notwithstanding, there are many ways in which these two classes are similar yet subtly different. The aim of this paper is to use martingales to make the reader believe this. For this reason, part of this paper is expository and part is not. Sometimes proofs will be included and sometimes not. It is now time to fix some terminology.

Let us agree that a Banach space is an Asplund space if each of its separable subspaces has a separable dual. A Banach space contains no copy of $l_1$ if it has no subspace linearly homeomorphic to the usual sequence space $l_1$.

Basic to this paper are two theorems. The first is due to Rosenthal [17] (and Dor [3] for the complex case).

**ROSENTHAL'S THEOREM.** A Banach space $X$ contains no copy of $l_1$ if and only if every bounded sequence in $X$ has a weakly Cauchy subsequence.

The second is due to Pełczyński [15]. We shall include a proof here because we think our proof is even easier than the proof indicated by Haydon in [9]. Naturally it depends on Rosenthal's fundamental theorem.
PEŁCZYŃSKI'S THEOREM. Any one of the following statements about a Banach space $X$ implies all the others:

(a) The space $X$ contains no copy of $\ell_1$.
(b) Every bounded linear operator from $L_1[0,1]$ into $X^*$ takes weakly compact sets into norm compact sets.
(c) The dual $X^*$ contains no copy of $L_1[0,1]$.

Proof. To prove that (a) implies (b), let $T: L_1[0,1] \to X^*$ be a bounded linear operator. Because $L_1[0,1]$ is separable, the closure of the range of $T$ is a separable subspace $Z$ of $X^*$. An appeal to a standard trick of Dunford and Schwartz's (see [5, V. 8.8] or [2, II. 3.6]) produces a separable subspace $Y$ of $X$ such that $Z$ is isometric to a subspace of $Y^*$. Note that $Y$ contains no copy of $\ell_1$. Thus we can assume that $T$ is a bounded linear operator from $L_1[0,1]$ into $Y^*$. Next the separability of $Y$ and an easy compactness argument originally due to Dunford and Pettis [4] (see [3, VI. 8.6] or the first part of the proof of [2, III. 3.1]) produces a bounded function $g: [0,1] \to Y^*$ such that

$$T(f)(y) = \int_{[0,1]} f(t)g(t)(y)dt$$

for all $f$ in $L_1[0,1]$ and for all $y$ in $Y$. To show $T$ maps weakly compact sets into norm compact sets, it is enough to show that $T$ acts as a (norm) compact operator from $L_1[0,1]$ into $Y$. (This is a very simple consequence of the facts that weakly compact sets in $L_1[0,1]$ are uniformly integrable and that uniformly integrable subsets of $L_1[0,1]$ can be uniformly approximated in the $L_1[0,1]$ norm by $L_\infty[0,1]$ bounded sets.)

To this end, define an operator $S: Y \to L_1[0,1]$ by

$$S(y)(t) = g(t)y.$$ 

For $y$ in $Y$ and $t$ in $[0,1]$. Let $(y_n)$ be a bounded sequence in $Y$. Since $Y$ contains no copy of $\ell_1$, Rosenthal's theorem guarantees that $(y_n)$ has a weakly Cauchy subsequence $(y_{n_j})$. A glance at the definition of $S$ shows that $S(y_{n_j})$ is an almost everywhere Cauchy sequence and hence converges almost everywhere to a measurable function $\phi$. Moreover the boundedness of $g$ and the boundedness of $(y_n)$ guarantee that $(S(y_{n_j}))$ is $L_\infty[0,1]$-bounded. This combined with the bounded convergence theorem proves that