This paper is concerned with the totally transcendental first-order theories and the notions of rank and degree introduced by Morley [10]. In the terminology of Shelah such theories are just those which are \( \mathcal{K}_0 \)-stable. From Marsh [9] and later work of Baldwin and the author [3] it is clear that the idea of the dimension of a model or of a formula in a model is one that is useful for analysing the structure of models of totally transcendental theories. A set \( A \) in a model \( M \) is called independent if for any distinct members \( a, b \) of \( A \) and formula \( \varphi(x,\overline{y}) \), \( M \models \varphi(a,\overline{b}) \) implies that the formula \( \varphi(x,\overline{y}) \) has the same rank as the universe, i.e. as \( x = x \). A model can be said to have dimension if all maximal independent sets have the same cardinality. Our main result is that every model of rank 1 or of rank 2 and degree 1 has dimension but that for models of higher rank or of rank 2 and degree > 1 there may be maximal independent sets of different cardinalities.

As a byproduct of our investigation of dimension we provide an analysis of the structure of \( \mathcal{K}_0 \)-stable theories for which the rank of the universe is 2 and the degree 1. Roughly speaking we can divide such theories into four groups distinguished as follows:

(i) there exists a sequence \( \langle \varphi_i(x) : i < \omega \rangle \) of formulas of rank 1 such that for every rank 1 formula \( \psi(x,\overline{a}) \) there exists \( n \) such that \( \psi(x,\overline{a}) \land \neg \varphi_n(x) \) is finite

(ii) there exists a formula \( \varphi(x) \) of rank 2 such that the theory
restricted to $\phi(x)$ is $\mathcal{K}_1$-categorical; in this case we call $\phi(x)$ an $\mathcal{K}_1$-categorical formula.

(iii) there exists no $\mathcal{K}_1$-categorical formula of rank 2 but there exists a definable equivalence relation which has infinitely many infinite equivalence classes.

(iv) there is no $\mathcal{K}_1$-categorical formula of rank 2 but there exists a definable relation which is expressible uniquely as the disjunction of two equivalence relations, each having infinitely many infinite equivalence classes, and in some model is an automorphism taking one equivalence relation onto the other.

Examples of these various kinds of theories are as follows. For (i) take the theory of countably many infinite disjoint unary relations. For (ii) we can take any disjoint union of a theory having rank 1 and a theory which is $\mathcal{K}_1$-categorical of rank 2 and degree 1. Specifically we could take $\text{Th}(M)$ where $|M| = \omega \cup (\omega \times \omega)$ and there are two unary function symbols $F$ and $G$ defined on $M$ by

$$F(i) = G(i) = i, \quad F(\langle i, j \rangle) = G(\langle j, i \rangle) = \langle i, i \rangle$$

for all $i, j < \omega$. For (iii) we can take the theory of an equivalence relation having infinitely many infinite equivalence classes. For (iv) we can take $\text{Th}(\langle \omega \times \omega ; R \rangle)$ where $R$ is the binary relation defined by

$$\langle i, j \rangle R \langle m, n \rangle \iff i = m \lor j = n$$

for all $i, j, m, n < \omega$.

We shall also be able to indicate what spectra can be associated with the theories under consideration. For instance a theory falling under (i) either has $|\alpha|$ models in every power $\mathcal{K}_\alpha > \mathcal{K}_\omega$ or $|\alpha|^\omega$.

A theory falling under (ii) is either $\mathcal{K}_1$-categorical or has $|\alpha|$ models in every power $\mathcal{K}_\alpha > \mathcal{K}_\omega$. A theory satisfying (iv) also has $|\alpha|$ models in $\mathcal{K}_\alpha$ for $\alpha > \omega$. The situation for theories of type (iii) is more complicated but here again we shall be able to delineate all the possibilities.

The notions of dimension, rank, and degree may seem at first glance extremely technical and not worthy of investigation in their own right. However, in the right context their role is extremely natural. For instance if we choose the right examples our dimension can become either the dimension of a vector space over the rationals or the cardinality of a transcendence basis for an algebraically