1.- INTRODUCTION.

The history of analytic spaces modelled on locally convex vector spaces began with DOUADY's celebrated thesis [4]. He introduced his notion of Banach-analytic spaces in order to solve a modul problem in finite dimensional complex analysis. Books which treat the local theory of these analytic spaces in its own right are RAMIS [13], for the Banach-analytic case, and, quite recently, MAZET [10], for analytic sets in locally convex vector spaces, respectively.

Global theory was developed only in the smooth case, e.g. by SCHOTTENLOHER and others (see NOVERRAZ [11]). But the field suffered (and still is suffering) from a lack of applications.

Already DOUADY observed how pathological Banach-analytic spaces may look like. During the past years, it has turned out that function theory behaves much better on the class of those analytic spaces which can be modelled on strong duals of nuclear (F)-spaces (= (DFN)-spaces). This is a very contrary class since the intersection of Banach-analytic and (DFN)-analytic spaces consists exactly of the class of finite dimensional complex spaces (cf. [8], Satz(3.1)).

First of all, the theorem of BOLAND/WAELBROECK [16] states that the algebra $\mathcal{O}(U)$ of all holomorphic functions on an open set $U$ in some linear (DFN)-space is a uniform nuclear (F)-algebra (with respect to the topology of compact convergence); thus the section algebra $H^0(X,\mathcal{O})$ of a (DFN)-analytic space $(X,\mathcal{O})$ is also a nuclear (F)-algebra (not necessarily uniform).

Rather recently, RABOIN's work [12] and a slight completion by COLOMBEAU/PERROT [2] opened the door to function theory on linear (DFN)-spaces. They showed that the $\bar{\partial}$-problem has solutions on pseudo-convex open subsets $\Omega$ of a linear (DFN)-space $E$ and, as one consequence among others, the vanishing
of the \(\check{\text{ech}}\) cohomology groups \(H^1(\Omega, \Theta_E)\). It is unknown, however, if also \(H^1(\Omega, \mathfrak{F})\) vanishes for ideal sheaves \(\mathfrak{F} \subset \Theta_E\) which are associated to certain analytic subsets.

In [8] we gave a rather simple characterization of Stein algebras (: = algebras of all holomorphic functions on Stein spaces). The principle of local spectralizing for \((\mathcal{O}, \text{FN})\)-analytic spaces (3.4) yields a characterization of finite dimensional complex spaces within the category of \((\mathcal{O}, \text{FN})\)-analytic spaces, by two simple properties (3.2): a \((\mathcal{O}, \text{FN})\)-analytic space \((X, \Theta)\) is a finite dimensional complex space if and only if

(i) \(X\) is locally compact,
(ii) \(\Theta\) is a uniform sheaf.

The principle of local spectralizing was announced in [8]; it will be proved in (3.4). It asserts that all reduced \((\mathcal{O}, \text{FN})\)-analytic spaces can locally be represented as spectra of some nuclear \((F)\)-algebra \(A\). Such spectra carry a canonical \((\mathcal{O}, \text{FN})\)-analytic structure denoted by \((\Theta A, \Theta A/\mathfrak{F}_0)\) (see (3.1)). Thus in a sense, the local study of \((\mathcal{O}, \text{FN})\)-analytic spaces is equivalent to the study of nuclear \((F)\)-algebras.

In section 4, we characterize the vanishing of the cohomology group \(H^1(A', \mathfrak{F})\), where \(\mathfrak{F}\) denotes the ideal sheaf of \(\Theta A\) in \(A'\). We suggest a universal holomorphic functional calculus (= h.f.c.) \(h: \Theta(\Theta A) \to A\), such that \(\hat{a} \mapsto a\), for all \(a \in A\); it contains all finite dimensional h.f.c. \(\Theta(\Theta(a_1, \ldots, a_n)) \to \Theta(\Theta A) \to A\) in the usual unique way. It is strictly larger than CRAW's h.f.c. [3].

A first step is an "entire" h.f.c.

\[
\Theta(A') \to A, \quad \text{s.t.} \quad \hat{a} \to a, \quad \forall a \in A,
\]

which always exists (4.1). The "universal" h.f.c. \(h: \Theta(\Theta A) \to A\) exists if and only if \(H^1(A', \mathfrak{F}) = 0\). Moreover, this is also equivalent to the vanishing of the cohomology groups \(H^1(\Omega, \mathfrak{F})\), for all pseudo-convex open neighbourhoods \(\Omega \subset A'\) of \(\Theta A\).

For example, all holomorphic algebras \(H^0(X, \Theta)\) corresponding with a complex space \((X, \Theta)\) enjoy this h.f.c. More generally, it is true for all sheaf-algebras (i.e. algebras of \(A\)-holomorphic functions in the sense of RICKART, see (2.3)). In fact, I don't know any nuclear \((F)\)-algebra without this h.f.c.. But counterexamples can easily be found (4.7) when the nuclearity condition is dropped.