1. Shape theory started as an organized branch of topology in 1968 with a series of papers by Borsuk opening with [1]. The basic idea was to modify homotopy so as to make it better suited to the study of spaces more general than those usually considered in homotopy theory (CW-complexes, ANR's). Borsuk actually defined a shape category of compact metric spaces embedded in the Hilbert cube \(\mathbb{Q}\) by replacing homotopy classes of maps \(X \rightarrow Y\) by homotopy classes of certain sequences of maps \(\mathbb{Q} \rightarrow \mathbb{Q}\), called fundamental sequences. It soon became clear that one can describe this category very efficiently by using inverse sequences of compact ANR's (Mardešić and Segal, [14], 1973). One expresses \(X\) and \(Y\) as limits of such sequences \(X\) and \(Y\) respectively. The role of a homotopy class of fundamental sequences \(X \rightarrow Y\) is then assumed by a morphism \(X \rightarrow Y\) of the procategory \(\text{pro-HTop}\) associated with the homotopy category \(\text{HTop}\).

It took several years before one was able to define shape morphisms between arbitrary topological spaces (Mardešić [12], 1973, Morita [17], 1975). Instead of inverse sequences in \(\text{Top}\) of compact ANR's one considers inverse systems in \(\text{HTop}\) of metric ANR's indexed by arbitrary directed sets. Instead of requiring that \(X\) is an inverse limit of \(X\), one requires that \(X\) is associated with \(X\) by means of a morphism of \(\text{pro-HTop}\) \(p:X \rightarrow \overline{X}\), which satisfies the following condition of Morita. For any ANR-system \(Y \in \text{pro-HTop}\) and any morphism \(h:X \rightarrow Y\) of \(\text{pro-HTop}\), there exists a unique morphism \(f:X \rightarrow Y\) of \(\text{pro-HTop}\) such that \(h = f \circ p\). One defines also a functor \(S: \text{HTop} \rightarrow \text{Sh}\), called the shape functor. It shows that shape gives a coarser classification of spaces than the homotopy type. However, for CW-complexes and ANR’s shape and homotopy type coincide.

Although shape theory was quite successful (see, e.g. Ch III of the book Mardešić and Segal, [15]), there were circumstances when one could not perform with shape some desired constructions. Thus arose the need for a strong shape theory, which would be closer to homotopy
than the ordinary shape. One can roughly compare the situation with
the one in homology, where one has singular, Čech and also Steenrod-
Sitnikov groups. The first two homologies correspond to homotopy and
ordinary shape and the last one corresponds to strong shape.

Strong shape theories for compact metric spaces were defined suc-
cessively by J.B.Quigley [18], D.A.Edwards and H.M.Hastings [8],
F.W.Bauer [1], Ju.T.Lisica [9], Y.Kodama and J.Ono [8], J.Dydak and
J.Segal [6], A.Calder and H.M.Hastings [3] and F.W.Cathey [4]. Altho-
ugh the approaches are quite different, it is known today that they all
yield isomorphic categories.

It is much more difficult to define a strong shape category SSh
for arbitrary topological spaces. One encounters serious difficulties
already in the special cases of compact Hausdorff spaces or separable
metric spaces. The first such theories are due to Bauer [1] and Edwards
and Hastings [7]. Other theories were developed by Lisica [10],
Z.R.Miminošvili [16] and more recently by Cathey and Segal [5]. The
mutual relationship of these theories has not yet been fully clarified.
The categories of Bauer, Lisica and Miminošvili are isomorphic and
probably differ from the ones of Edwards-Hastings and Cathey-Segal.

The main purpose of this paper is to give a new definition of the
strong shape category SSh of topological spaces. The special feature
and advantage of our definition consists in the fact that it is con-
structive and analogous to Morita’s definition of the ordinary shape
category Sh. The role of the category pro-HTop is here taken by a
coherent prohomotopy category CPHTop, which we also define. Instead
of morphisms $p:X \to X$ satisfying Morita’s condition, we use here reso-
lutions, a new tool developed by S.Mardešić in [13]. We also define a
strong shape functor $S_1: HTop \to SSh$ and a forgetful functor $S_2: SSh \to Sh$
such that the usual shape functor $S: HTop \to Sh$ is the composition
$S = S_2S_1$. Consequently, strong shape lies between homotopy and ordinary
shape. We believe that our strong shape category is isomorphic to the
category of Edwards and Hastings.

[11] is a detailed version of the present paper containing all the
proofs.

2. Let $\Delta^n \subseteq \mathbb{R}^{n+1}$ be the standard $n$-simplex. If $n > 0$ and
$0 \leq j \leq n$, we denote by $\partial_j^n: \Delta^{n-1} \to \Delta^n$ the $j$-th face operator,
given by

$$\partial_j^n(t_0, \ldots, t_{n-1}) = (t_0, \ldots, t_{j-1}, 0, t_j, \ldots, t_{n-1}).$$