Third Family of $N = 2$ Supersymmetric KdV Hierarchies

S. Krivonos and A. Sorin

Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow Region, Russia

Abstract. We propose the Lax operators for $N = 2$ supersymmetric matrix generalization of the bosonic $(1, s)$-KdV hierarchies. The simplest examples – the $N = 2$ supersymmetric $a = 4$ KdV and $a = 5/2$ Boussinesq hierarchies – are discussed in detail.

1 Introduction

The existence of three different infinite families of $N = 2$ supersymmetric integrable hierarchies with the $N = 2$ super $W_s$ algebras as their second Hamiltonian structure is a well-established fact by now (Labelle and Mathieu (1991), Yung (1993), Yung and Warner (1993)). Their bosonic limits have been analyzed in (Bonora, Krivonos and Sorin (1996)), and three different families of the corresponding bosonic hierarchies and their Lax operators have been selected. Then, a complete description in terms of super Lax operators for two out of three families has been proposed in (Delduc and Gallot (1997), Bonora, Krivonos and Sofia (1998)), and the generalization to the matrix case has been derived in (Bonora, Krivonos and Sorin (1998)).

The last remaining family of $N = 2$ hierarchies is supersymmetrization of the bosonic $(1, s)$-KdV hierarchies (Bonora, Krivonos and Sorin (1996)). We call them the $N = 2$ supersymmetric $(1, s)$-KdV hierarchies. As opposed to the bosonic counterparts of the former two hierarchies (Bonora, Krivonos and Sorin (1996)), the $(1, s)$-KdV hierarchy is irreducible (see (Bonora, Liu and Xiong (1996)) and references therein), i.e. its Lax operator cannot be decomposed into a direct sum of some more elementary components. This reduction property leads to a strong restriction of the original supersymmetric Lax operator: its bosonic limit should be irreducible. In other words, it should generate only a single operator component. This property is surely satisfied for a supersymmetric Lax operator which is a pure bosonic pseudo-differential operator with the coefficients expressed in terms of $N = 2$ superfields and their fermionic derivatives in such a way that it commutes with one of the two $N = 2$ fermionic derivatives. The Lax operator of this kind has in fact been observed in (Bonora, Krivonos and Sorin (1996)) for the $N = 2 a = 5/2$ Boussinesq hierarchy in the negative-power decomposition over bosonic derivative up to the $\delta^{-5}$ order. Quite recently, its closed analytic representation has been obtained in (Gribanov, Krivonos and Sorin (1998)).
The aim of the present letter is to present a new infinite class of reductions (with a finite number of fields) of $N = 2$ supersymmetric matrix KP hierarchy which includes the above-mentioned family of $N = 2 (1, s)$-KdV hierarchies in the scalar case.

2 Extended Matrix $N = 2$ Super $(1, s)$-KdV Hierarchy

The Lax operator

$$L_{KP}^{red} = I \partial + a_0 + \omega_0 D + \sum_{j=-\infty}^{-1} \left( a_j \partial - [D a_j] D + \omega_j D \partial - \frac{1}{2} [D \omega_j] [D, D] \right) \partial^{j-1}$$

(1)
derived by reduction $[D, L_{KP}^{red}] = 0$ (Sorin (1997)) of the $N = 2$ supersymmetric matrix KP hierarchy has been constructed in (Bonora, Krivonos and Sorin (1998)). Here, $a_j$ and $\omega_j$ at $j \geq 1$ ($a_0$ and $\omega_0$) are generic (chiral) bosonic and fermionic square matrix $N = 2$ superfields. The Lax operator (1) still contains an infinite number of fields. Its further reductions (Bonora, Krivonos and Sorin (1998)),

$$L_{KP}^{red} = I \partial + \sum_{j=1}^{s-1} \left( J_{s-j} \partial - [D J_{s-j}] D \right) \partial^{j-1} - J_s - D \partial^{-1} [D J_s] - \mathcal{F} \bar{\mathcal{F}} - \mathcal{F} \bar{D} \partial^{-1} [D \mathcal{F}],$$

(2)

are characterized by a finite number of fields and contain two out of three families of $N = 2$ supersymmetric hierarchies with $N = 2$ super $W_s$ algebras as their second Hamiltonian structure in the scalar case at $\mathcal{F} = \bar{\mathcal{F}} = 0$ (see (Bonora, Krivonos and Sorin (1998)) for details).

It appears that besides reductions (2), there exist other reductions of the Lax operator (1) which in the scalar case correspond to the last remaining family of $N = 2$ hierarchies with the $N = 2$ super $W_s$ algebras as their second Hamiltonian structure, i.e. $N = 2 (1, s)$-KdV hierarchies. Based on the inputs given above, we are led to the following conjecture for the expression of the matrix-valued pseudo-differential operator with a finite number of superfields representing the new reductions of the Lax operator (1):

$$L_{KP}^{red} = L_s = I \partial - [D L_s^{-1} \bar{D} L_s], \quad L_s = I \partial^s + \sum_{j=0}^{s-1} J_{s-j} \partial^j + \bar{\mathcal{F}} \partial^{-1} \mathcal{F}$$

(3)

Here, $s = 0, 1, 2, \ldots, F \equiv F_{aA}(Z)$ and $\mathcal{F} \equiv \bar{F}_{Aa}(Z)$ ($A, B = 1, \ldots, k; a, b = 1, \ldots, n + m$) are chiral and antichiral rectangular matrix-valued $N = 2$ superfields,

$$DF = 0, \quad \bar{D} \mathcal{F} = 0,$$

(4)