Abstract: A one-dimensional fermion model with bond charge-interactions as well as Hubbard-type interactions is investigated exactly. The large distance asymptotics of the density-density and pair correlations are calculated. The system shows Luther-Emery liquid behaviour with a crossover from a density-density dominated regime to one with dominant pair correlations. Furthermore, the integrable $tJ$ chain is studied at finite temperatures. Some concepts for this analysis are introduced, notably the Trotter-Suzuki mapping and the quantum transfer matrix. Finally, the specific heat is presented and its structure discussed.

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1 Introduction

In this contribution to the Winterschool, I will give a short introduction into the physics of one-dimensional quantum chains. I will focus on the study of integrable systems which succumb to exact treatments, notably by the Bethe ansatz. Quite generally integrability imposes strong constraints on the interaction terms. Nevertheless, the physical properties of such systems appear to be generic and quite representative of other non-integrable systems.

The most famous examples of integrable models are the spin-1/2 Heisenberg chain and the 1d Hubbard model [1, 2, 3]. Here, however, I want to discuss fermionic systems with additional interactions different from the on-site Coulomb repulsion. In the course of the search for purely electronic mechanisms for high-$T_c$ superconductivity a generalized Hubbard model was derived as a more detailed tight-binding description of the electronic system of solid matter

$$H = - \sum_{\langle i,j \rangle} t_{ij} \left( c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right) + U \sum_i n_{i\uparrow} n_{i\downarrow} + V \sum_{\langle i,j \rangle} n_{i} n_{j},$$  

with $U$ the on-site Hubbard term, $V$ the nearest-neighbour Coulomb interaction and the hopping integral

$$t_{ij} = t_0 - \Delta t(n_{i,-\sigma} + n_{j,-\sigma}).$$
Such correlated hopping terms (bond-charge interactions) arise from overlap integrals which are different for singly occupied orbitals than for multiply occupied orbitals.

For repulsive electrons, the model parameters in Eq. (1) should be positive $t_0, U, V > 0$. In particular $\Delta t > 0$ leads to a suppression of the hopping of a particle between two sites if one of them is already occupied by an electron (with opposite spin). However, the nature of all of these apparently repulsive interaction terms is different as revealed by the particle hole transformation and sublattice phase shift, $c_{i\sigma} \rightarrow \pm c_{i\sigma}^*$ [4, 5]. Neglecting a chemical potential term, the result on Eq. (1) is a transformation of

$$U, V, t_0 \rightarrow +U, +V, +(t_0 - 2\Delta t), \quad \Delta t \rightarrow -\Delta t.$$  

Thus, the Hubbard terms $U$ and $V$ are repulsive for holes as for electrons, the correlated hopping term, however, becomes attractive for holes. This observation was a cornerstone of the concept of hole superconductivity in Refs. [4, 5]. For mean-field and Lanczos studies of Eq. (1) the reader is referred to Ref. [6]. We are interested in exact results for Eq. (1) obtained in 1d. Unfortunately, even the dimensional restriction does not generally guarantee exact solutions. For Eq. (1), only the special case of the standard Hubbard model is integrable. On the other hand, we may add certain interaction terms which keep the physics of Eq. (1), but render the mathematics more tractable. If we consider a one-dimensional Hamiltonian with correlated hopping terms, Hubbard on-site interaction and pair hopping processes

$$H = -\sum_{j,\sigma} (c_{j+1,\sigma}^+ c_{j+1,\sigma} + c_{j+1,\sigma}^+ c_{j,\sigma}) \exp \left[ -\frac{1}{2} (\eta - \sigma \gamma) n_{j+1,\sigma} - \frac{1}{2} (\eta + \sigma \gamma) n_{j+1,-\sigma} \right] + \sum_j \left[ U n_{j-1,n_j} + t_p \left( c_{j-1,j}^+ c_{j+1,j} c_{j+1,j+1} + h.c. \right) \right],$$  

we have integrability under the condition [7]

$$t_p = U/2 = \pm \left[ 2e^{-\eta} (\cosh \eta - \cosh \gamma) \right]^{1/2},$$  

where we restrict ourselves to the physical positive sign in the following. As a criterion of integrability we may either use the condition of factorizing $S$-matrices [8] or the existence of a classical model satisfying the Yang-Baxter equation [3] and leading to Eq. (4) in the Hamiltonian limit.

The system Eq. (4) shows an apparent left-right asymmetry in the correlated hopping term, which however does not affect transport properties of the system. In fact, no charge or spin current is imposed by the asymmetry. The asymmetry seems to be irrelevant from the physical point of view. Mathematically, however, it is essential for integrability and the analytical study.

The second system I will consider is the one-dimensional $tJ$ model describing hopping of electrons from singly occupied lattice sites to empty sites and a Heisenberg like spin exchange for nearest-neighbours

$$H = -t \sum_{j,\sigma} \mathcal{P}(c_{j,\sigma}^+ c_{j+1,\sigma} + c_{j+1,\sigma}^+ c_{j,\sigma}) \mathcal{P} + J \sum_j (S_j S_{j+1} - n_j n_{j+1}/4).$$