Efficient calculation
of 3-D turbulent transonic flows

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Abstract. A second-order scheme designed to compute 3-D viscous flows is presented: it is based on a Lax-Wendroff solver modified by a characteristic time-step technique. The scheme is driven to steady-state by various implicit treatments, the pros and cons of which are discussed in terms of cost and memory requirement. The efficiency and accuracy of the proposed methods are assessed by computing transonic flows over wings.

1 Introduction

An implicit centred method of the Lax-Wendroff type was originally proposed in the early eighties to solve the multidimensional Euler equations with second-order accuracy [Lerat (95)]. Lately, this method was modified by introducing a characteristic time-step matrix in the numerical Lax-Wendroff fluxes. The resulting scheme retains the following properties of the original scheme: simplicity, true second-order accuracy at steady-state in $d$ dimensions, compactness ($1 + 2d^2$ points), no tuning parameter (no artificial viscosity, limiters or correction), unconditional linear stability, low internal dissipation; moreover it adds new interesting features: steady solution independent of the CFL number, quicker convergence to the steady-state, higher robustness.

In one dimension, the new scheme is purely upwind. In several dimensions, it is close to and less dissipative than a genuinely multidimensional upwind scheme [Huang, Lerat (98)]. This method has been extended to the compressible Navier-Stokes equations [Corre, Huang, Lerat (97)] and applied to transonic flows in 2-D turbulent and 3-D laminar regimes using block line-relaxation techniques.

In the present paper, we carry on the assessment of the method for 3-D laminar and turbulent transonic flows over wings and discuss a crucial issue for complex aerodynamic configurations: the proper balance between fast convergence and low memory requirements.
2 Characteristic time-step and Lax-Wendroff solver

2.1 Euler equations

Let us consider the Euler equations in $d$ space-dimensions:

$$\frac{\partial w}{\partial t} + \sum_{p=1}^{d} \frac{\partial f_p}{\partial x_p} = 0 \, ,$$

(1)

where $f_p = f_p(w)$. The semi-discrete Lax-Wendroff approximation of (1) is:

$$\frac{w^{n+1} - w^n}{\Delta t} + \sum_{p=1}^{d} \frac{\partial F_p}{\partial x_p} = 0 \, ,$$

(2)

where $w^n$ denotes the numerical solution at time $n\Delta t$ and $F_p$ is:

$$F_p = f_p^n - \frac{\Delta t}{2} A_p^n \sum_{q=1}^{d} \frac{\partial f_q^n}{\partial x_q} \, ,$$

(3)

with the flux Jacobian matrix $A_p = df_p/dw$. In order to suppress the dependence on the time-step of the steady solution, $\Delta t$ may be replaced in (3) by a matricial time-step $\Delta t_p$, the characteristic time-step, defined by $\frac{\Delta t_p}{\delta x_p} | A_p^n | = I$, where $\delta x_p$ is the space increment in the $x_p$ direction, $I$ is the identity matrix and, in fully discrete form, $A_p$ stands for a Roe average. To ensure the stability of the resulting upwind scheme, it is necessary to use a unique time-step $\Delta t'$. In the scalar case, an optimal choice was found to be the minimal characteristic time-step $\Delta t' = \min_p (\frac{\delta x_p}{| A_p |})$; after substitution in (3), one obtains the following scheme, originally proposed by [Huang, Lerat (98)] and denoted HL scheme from now on:

$$\frac{w^{n+1} - w^n}{\Delta t} + \sum_{p=1}^{d} \frac{\partial f_p^n}{\partial x_p} = \sum_{p=1}^{d} \delta x_p \frac{\partial}{\partial x_p} [\Phi_p \text{sgn}(A_p^n) \sum_{q=1}^{d} \frac{\partial f_q^n}{\partial x_q}] \, ,$$

(4)

with $\Phi_p = \min_{q=1,d} (1, \frac{\delta x_q}{\delta x_p} | A_p |)$. When applied to the solution of the Euler equations, the HL scheme retains the formal expression (4) with the following adaptation in the definition of the now matricial coefficient $\Phi_p$. Let $\lambda_p^{(k)}$ be the $k^{th}$ eigenvalue of $A_p$ and $P_p$ be a regular matrix having linearly independent eigenvectors of $A_p$ as column vectors; $\Phi_p$ is then defined as the matrix $\Phi_p = P_p \text{diag}(\phi_p^{(k)}) P_p^{-1}$, with $\phi_p^{(k)} = \min_{q=1,d} (1, \frac{\delta x_q}{\delta x_p} | \lambda_p^{(k)} |)$, where $m(A_q) = \min_k (| \lambda_q^{(k)} |)$. The space discretization of the scheme is purely centred and makes use of a predictor-step for each space-direction, following [Lerat (95)]. The ability of the HL scheme to provide accurate steady solutions for a wide range of 2-D inviscid flows was demonstrated in [Huang, Lerat (98)].