Numerical Study of Effects of a Bellmouth on the Entrance Pipe Flow

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Abstract. The flow characteristics in the bellmouth region of a circular pipe have been numerically studied for Reynolds numbers (Re) ranging from 1000 to 4000. Until now, the axial velocity has been assumed to be uniform at the bellmouth outlet. The calculation results, however, reveal that it develops a profile somewhat towards the Poiseuille's parabolic profile. Moreover, a feasible reason for the increase in critical Reynolds number when using bellmouths was identified, i.e., if the bellmouth region is replaced by the pipe one, it was observed that the equivalent Re decreases from the original Re value.

1 Introduction

Most experimental studies of laminar incompressible fluid flow in the entrance region of a smooth circular pipe have been carried out using bellmouth entrances. In theoretical analyses, since bellmouth entrances involve complex geometries, it is difficult to solve the full Navier-Stokes equations.

Therefore, the primary objective of this investigation is to numerically study the effects of a bellmouth on the entrance flow: (i) velocity and vorticity distributions and (ii) radial pressure distributions.

Bellmouths are designed to show the following effects on the entrance flow: (a) The entrance to the pipe is well rounded, and the fluid enters smoothly from a reservoir, having almost uniform velocity and pressure distributions over the pipe inlet cross section. (b) The fluid will always carry some residual disturbances along with it. Bellmouths are used to minimize disturbances prior to flow entering the pipe; with smaller disturbances, laminar flow will persist to higher Reynolds numbers.

2 Numerical Calculations

2.1 Basic Equations

Unsteady flow of an incompressible Newtonian fluid with constant viscosity and density is considered. We first introduce the streamfunction, $\psi$, and vorticity, $\omega$, for the governing equations to avoid the explicit appearance of pressure, $p$, and later calculate the pressure distribution. The transport
equation for vorticity and the pressure in the Poison form are expressed, in dimensionless form, as follows [Kanda (98)]:

\[
\frac{\partial \omega}{\partial t} - \frac{1}{r} \frac{\partial \psi}{\partial z} \frac{\partial \omega}{\partial z} + \frac{1}{r} \frac{\partial \psi}{\partial r} \frac{\partial \omega}{\partial r} + \frac{\omega}{r^2} \frac{\partial \psi}{\partial z} = \frac{1}{Re} \left[ \frac{\partial}{\partial r} \left\{ \frac{1}{r} \frac{\partial}{\partial r} (r \omega) \right\} + \frac{\partial^2 \omega}{\partial z^2} \right] \quad (1)
\]

\[
\frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \frac{\partial}{\partial z} \left( r \frac{\partial p}{\partial z} \right) = -2 \left[ r \left( \frac{\partial v}{\partial r} \right)^2 + 2 \frac{\partial u}{\partial r} \frac{\partial v}{\partial z} + \left( \frac{\partial u}{\partial z} \right)^2 + \frac{v^2}{r} \right] \quad (2)
\]

The boundary condition for pressure is expressed as

\[
\left. \frac{\partial p}{\partial r} \right|_{r=wall} = -\frac{2}{Re} \left[ \nabla \times \omega \right]_r \quad \left. \frac{\partial \omega}{\partial z} \right|_{r=wall} = \frac{2}{Re} \left. \frac{\partial \omega}{\partial z} \right|_{r=wall} \quad (3)
\]

### 2.2 Transformation of Coordinates

An algebraic grid generation method is applied for transformation from the cylindrical \((z, r)\) to curvilinear coordinates \((\xi, \eta)\), since the pipe flow can be assumed to be axisymmetric in two dimensions. The first- and second-order derivatives are expressed as

\[
\left( \begin{array}{c}
\frac{\partial f}{\partial z} \\
\frac{\partial f}{\partial r}
\end{array} \right) = \frac{1}{J} \left( \begin{array}{cc}
r_\eta & -r_\xi \\
-z_\eta & z_\xi
\end{array} \right) \left( \begin{array}{c}
\frac{\partial f}{\partial \xi} \\
\frac{\partial f}{\partial \eta}
\end{array} \right), \quad J = z_\xi r_\eta - z_\eta r_\xi \quad (4)
\]

\[
\frac{\partial^2 f}{\partial z^2} = \frac{1}{J} [r_\eta (f_z)_\xi - r_\xi (f_z)_\eta], \quad \frac{\partial^2 f}{\partial r^2} = \frac{1}{J} [-z_\eta (f_r)_\xi + z_\xi (f_r)_\eta] \quad (5)
\]

where \(J\) is the Jacobian.

The bellmouth shapes usually depend on assumptions made by researchers. In this study the cross-section of the planar curve of the bellmouth surface, \(BW(z)\), is a circular arc as shown in Fig. 1: (i) radius at the bellmouth inlet, \(BR = D\), where \(D\) is the pipe diameter, (ii) axial length between the bellmouth inlet and its outlet, \(BZ = D\), and (iii) center and radius of the circle are \((D, 7D/4)\) and \(5D/4\), respectively. Hence the pipe wall is tangent to the circle at the pipe inlet. \(BW(z)\) is expressed nondimensionally as

\[
(z - 1)^2 + \left( BW(z) - \frac{7}{4} \right)^2 = \left( \frac{5}{4} \right)^2 \quad (6)
\]

The grid spaces, \(\Delta z\) and \(\Delta r\), and grid point, \((z, r)\), are written as

\[
\Delta z = \frac{BZ}{10 - 1}, \quad z(i) = (i - 1) \Delta z, \quad \Delta r = \frac{BW(z)}{J_0 - 1}, \quad r(i, j) = (j - 1) \Delta r \quad (7)
\]