Finite element method in the study of the multiple deck boundary layer flow of a class of non-Newtonian fluids

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**Abstract.** A numerical study of the boundary layer flow of several classes of non-newtonian fluids is presented. Several types of perturbations can be considered in a rigorous asymptotic analysis leading to a multiple-deck structure. Depending on the type of fluid and the shape of the profile, different types of equations are obtained. The complex boundary conditions including Hilbert integral matching conditions require the use of iterative procedure. The numerical method, available for a large class of flows, uses a finite element method over an adaptive grid. We present the results obtained for power-law and Carreau-Bird fluids flows over a hump.

1 Introduction

With the increase of the technological importance of non-Newtonian fluids in the last few decades there has been a great deal of work on the mathematical modeling of this type of flows. A special interest was focused on the boundary layer flows, trying to explain the important reduction of the laminar shear-stress experimentally observed for certain classes of fluids. The non-Newtonian character of the fluid strongly modifies the shape of the velocity and pressure profiles in the boundary layer.

The triple-deck boundary layer theory proved to be an interesting tool in numerical studies of flows over arbitrary profiles. It enables the analysis of one of the most important phenomena characterizing the aerodynamics of the profile: the development of reversed flows. Recently, the theoretical works of Mauss [Mauss (94)] offered a solid mathematical basis for the triple-deck and double-deck structures.

Following these results, a rigorous multiple-deck theory has been worked out for this class of incompressible non-Newtonian fluids [Andrei (97)]. A generalized Reynolds number is introduced using a reference viscosity. For a laminar flow at a high generalized Reynolds number, it has been proved that triple-deck and double-deck structures can be used for several types of fluids, including Carreau-Bird, Prandtl, White-Mesner as well as pseudo-plastic and power-law fluids.

Our numerical study deals with the inner boundary layer equations.
2 The Mathematical Model

2.1 The Physical Problem

In this paper we consider the steady laminar flow of a polymeric fluid within one of the above mentioned types over a hump or a series of humps on an otherwise flat plate.

The shear stress tensor, $\mathbf{T}$, and rate of strain tensor, $\mathbf{D}$, are related by the constitutive law : $\mathbf{S} = \eta(I_2)\mathbf{D}$ where $\eta(I_2)$, the function of viscosity, is an arbitrary continuous function of the second invariant of the tensor $\mathbf{D}$.

A lot of expressions are successfully used, in industrial applications, for the function $\eta$. In this study we only consider the case of functions admitting a series development in a vicinity of the origin, and fulfilling the condition :

$$\exists r, k \in \mathbb{R} \text{ finites such as } \lim_{z \to 0} \frac{\eta(x)}{x^r} = k.$$

We notice that all the fluids models above mentioned fulfill this condition.

We define a small parameter $\varepsilon$ such that $\varepsilon \mathbf{e} = \varepsilon^m$ where $\mathbf{e}$ is the generalized Reynolds number.

We assume that $\varepsilon^\alpha$ and $\varepsilon^\beta$ are respectively the scale perturbations in the longitudinal and in the transversal directions. Thus, the hump is defined by

$$y = \varepsilon^\beta f \left( \frac{x}{\varepsilon^\beta} \right)$$

where $f$ is a sufficiently smooth function.

2.2 Asymptotic modelling for separating boundary layers

Following the work of Mauss [Mauss (94)] we are looking for a triplet $(\alpha, \beta, r)$ for which a deck of the classical boundary layer is not strongly modified. We introduce therefore $\alpha$ : the scale of the induced perturbation of the longitudinal velocity.

In order to satisfy the boundary conditions we introduce an inner-layer in the vicinity of the plate. The solution of the equations obtained in the middle-layer does not fit with the exterior characteristics of the flow. We must therefore predict an exterior layer between the the middle-deck and the outer flow.

Using a Van-Dyke type asymptotic matching principle we obtain the scales of the decks as well as the range of the induced perturbation. This information enables us to write the equations in each of the three decks. We first obtain a general constraint for the perturbation scales : $\beta = \frac{\alpha}{3} + \frac{m}{2} - \frac{rm}{6}$.

The scale of the induced longitudinal perturbation, $\alpha$ and the range $\alpha$ are related by :