The Quantum Liquids $^3$He and $^4$He

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Summary: Recent progress in the microscopic theory of the helium liquids is reviewed. Particular emphasis is put on the idea of a unified description of Fermi and Bose liquids in terms of polarization potential theory. It is shown that the qualitative structure of the polarization potentials or equivalently of Landau's effective interaction follow from the assumption of a so-called direct interaction.

1 Introduction

The helium liquids have fascinated physicists for a long time [1, 2]. As far as we know, condensed helium under normal conditions remains in the liquid state down to absolute zero. This unusual property can be ascribed to a peculiar coincidence: the helium nuclear mass is just small enough for the zero point motion to overcome the van-der-Waals binding forces and to prohibit the atoms from forming a crystal lattice. More precisely it is the value of the product $\alpha (m_n/m_e)$ of the fine structure constant $\alpha$ and the ratio of nuclear mass $m_n$ to electron mass $m_e$ that is crucial here.

In the liquid state quantum exchange effects are much more important than in the solid state. Consequently, quantum coherence of the many-body state is enhanced at low temperatures, eventually giving rise to the collective motion of particles in a coherent quantum state, the superfluid state. In the case of the Bose liquid $^4$He the coherent state finds its counterpart in the Bose-Einstein condensed state of the non-interacting Bose gas. For the Fermi liquid $^3$He the situation is more complicated, since the corresponding non-interacting system, the Fermi gas, does not condense into a collective state. Here the fermions are required to form Cooper pairs first (which behave essentially as bosons) before a condensation in the superfluid state can take place. Since the attractive interaction responsible for the formation of pairs is weak, the transition temperature into the superfluid state of $^3$He, $T_c \approx 1...3$ mK, is three orders of magnitude below the degeneracy temperature $T_F \approx 1$ K.

The macroscopic properties of the helium liquids have been studied in great detail over the past four decades. Most of the phenomena observed are well accounted for in terms of thermodynamic and hydrodynamic theory and model theories like mean field theory, kinetic theory, the theory of critical phenomena and so forth.

Festkörperprobleme XXV (1985)
Interesting and important new aspects continue to emerge on both the experimental and theoretical sides [3]. However, during the time when all this wealth of knowledge on the macroscopic quantities has been accumulated, microscopic theory remained in a state of infancy. It is only since the early seventies that we slowly begin to understand better the gross features of the microscopic physics of liquid helium. This understanding takes place in several ways.

2 Microscopic Theory

First, there are now powerful Green's function Monte Carlo techniques available for integrating the many-particle Schrödinger equation. At present, there are results available on the ground state of the Bose system [4] and the ground state of the Fermi system [5]. There is hope that it will eventually be feasible to calculate finite temperature equilibrium properties or even transport properties using these methods.

A somewhat different line of approach is based on variational methods for the ground state energy [6] using Jastrow wave functions and cluster expansions like the hypernetted chain summation. The results of a generalized Jastrow-type variational calculation for the ground state energy of both $^4\text{He}$ and $^3\text{He}$ is in reasonable agreement with the exact Green's function Monte Carlo calculations [6]. A systematic improvement of the variational method is attempted within the correlated basis function (CBF) approach [7]. There the correlation operators determined in a variational calculation are used to construct a set of non-orthogonal basis states, on which a perturbation theory is built. This has the advantage of avoiding the complications stemming from the hard core part of the potential. The method provides good results for the ground state energies [6, 8]. It holds promise for a calculation of excited state properties [8]. However, those properties involving details of the excitation spectra, in particular of $^3\text{He}$ near the Fermi surface remain difficult to calculate.

At the present stage it is useful to consider also theories on a level which is between the fully microscopic and the semiclassical quasi-particle regimes. Model theories of this type may serve to bring out the salient properties of a many-body system on the microscopic scale, at the cost of introducing certain model parameters like an effective mass or an effective interaction function. The latter quantities may be more amenable to microscopic calculation than the directly observable quantities. It is this type of theory I would like to consider in more detail in the following.

3 Polarization Potential Theory

Any microscopic theory of liquid helium has to face two basic difficulties: Firstly, the radius of the repulsive hard core in the interatomic potential is large, up to 70% of the average interparticle distance. Consequently, there is a large steric hindrance effect present, which may be expected to increase the radius of the effective