Vision Guided Navigation for a Nonholonomic Mobile Robot

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1. Introduction

This contribution addresses the navigation task for a nonholonomic mobile robot tracking an arbitrarily shaped ground curve using vision sensor. We characterize the types of control schemes which can be achieved using only the quantities directly measurable in the image plane. The tracking problem is then formulated as one of controlling the shape of the curve in the image plane. We study the controllability of the system characterizing the dynamics of the image curve and show that the shape of the curve is controllable only up to its "linear" curvature parameters. We present stabilizing control laws for tracking both piecewise analytic curves and arbitrary curves. The observability of the curve dynamics is studied and an extended Kalman filter is proposed to dynamically estimate the image quantities needed for feedback controls. We concentrate on the kinematic models of the mobile base and comment on the applicability of the developed techniques for dynamic car models.

Control for steering along a curved road directly using the measurement of the projection of the road tangent and its optical flow has been previously considered by Raviv and Herman [9]. Stability and robustness issues have not been addressed, and no statements have been made as to what extent these cues are sufficient for general road scenarios. A visual servoing framework proposed in [2] addresses the control issues directly in the image plane and outlines the dynamics of certain simple geometric primitives (e.g. points, lines, circles). The curve tracking and estimation problem originally outlined in Dickmanns [1], has been generalized for arbitrarily shaped curves addressing both the estimation of the shape parameters as well as control in [3] by Frezza and Picci. They used an approximation of an arbitrary curve by a spline, and proposed a scheme for recursive estimation of shape parameters of the curve, and designed control laws for tracking the curve. For a theoretical treatment of the image based curve tracking problem, the understanding of the dynamics of the image of an arbitrary ground curve is crucial.
2. Curve Dynamics

In this section we derive image curve dynamics under the motion of a ground-based mobile robot. In the following, only the unicycle model is studied in detail. We will later comment on generalization to other mobile robot models.

Let \( p_{f_m} = (x, y, z)^T \in \mathbb{R}^3 \) be the position vector of the origin of the mobile frame \( F_m \) (attached to the unicycle) from the origin of a fixed spatial frame \( F_f \), and \( \theta \in \mathbb{R} \) be the rotation angle of \( F_m \) with respect to \( F_f \), defined in the counter-clockwise sense about the \( y \)-axis, as shown in Figure 2.1. For unicycle kinematics, one has:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = 
\begin{bmatrix}
\sin \phi & \cos \phi & 0 \\
-y \cos \phi & -x \sin \phi & -v \\
\end{bmatrix}
\begin{bmatrix}
0 \\
v \\
v \cos \phi
\end{bmatrix}
+ 
\begin{bmatrix}
v \sin \phi + z \cos \phi \\
-x \sin \phi \\
-x \cos \phi
\end{bmatrix}
\omega.
\]

From (2.1), the velocity of a point \( q \) attached to the camera frame \( F_c \) is given in the (instantaneous) camera frame by:

\[
\dot{p}_{f_m} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = 
\begin{pmatrix} v \sin \theta \\ 0 \\ v \cos \theta \end{pmatrix}, \quad \dot{\theta} = \omega
\]

where the steering input \( \omega \) controls the angular velocity; the driving input \( v \) controls the linear velocity along the direction of the wheel.

A camera with a unit focal length is mounted on the mobile robot facing downward with a tilt angle \( \phi > 0 \) and elevated above the ground by a distance \( d \), as shown in Figure 2.2. The camera coordinate frame \( F_c \) is such that the \( z \)-axis of \( F_c \) is the optical axis of the camera, the \( x \)-axis of \( F_c \) is that of \( F_m \), and the optical center of the camera coincides with the origins of \( F_m \) and \( F_c \).

In order to simplify the notation, we use the abbreviations \( s\phi, c\phi, ct\phi \) and \( t\phi \) to represent \( \sin \phi, \cos \phi, \cot \phi \) and \( \tan \phi \), respectively.