Iterative Model Based $H_2/H_\infty$ Synthesis for Active Suspension System

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1 Introduction

The basis of the model-based controller design scheme is an input-output model that usually derives from an identification process. Since the identified model tends to the actual plant more or less because of the modeling error and uncertainties, therefore the difference between the model and the plant has to be considered in the controller design process. On the other hand, the model has to be identified as accurately as possible in frequency domains that are important in the sense of the controlled system, and in other frequency domains accuracy is less important, so the control law has to be considered in the identification process. Consequently, one should identify the model for controller design and at the same time, one should consider the controller in the identification process. However, this is an impossible task because controller design depends on the identified model while model identification depends on the controller. The identification and the controller design are not independent during the design process, but they pertain to each other, so the design procedure should be performed in an iterative way. Recently, the iterative identification and controller design has come to the forefront in the field of control research. Its aim is to enhance the performance of the controller based on the identified model, which serves as a controller design step.

The idea of the iterative model-based controller design as a way of adaptive control theory has been formulated. In the adaptive scheme, the idea of updating the controller comes naturally, since the parameters of the model are revised in each time step. In the iterative scheme, updating of the controller is performed in an off-line way using the measured data, i.e. the performance of the controller is enhanced until it is possible [2,17]. The main paths of iterative approaches are
summarized in several important survey papers [9,23]. The iterative method proposed in this paper is based on the mixed $H_2/H_\infty$ control design. Two main directions are connected with this scheme, namely the Zang scheme and the three-stage scheme by Van den Hof and Schrama. The Zang scheme is based on the Linear Quadratic and Gaussian (LQG) optimization criterion with Least Squares (LS) identification [8,27]. The three-stage scheme is based on the $H_\infty$ norm controller design, closed loop identification method and robustness analysis [25].

This paper presents an iterative model-based controller synthesis to design a mixed $H_2/H_\infty$ controller. The mixed norm optimization results in a quadratic performance index that is close to the LQG performance index, and an $H_\infty$ norm that is close to the $H_\infty$ optimal solution. For practical application of the method, the iterative scheme will be demonstrated on an active suspension problem based on the so-called quarter-car model. The aim of the design is to find a stabilizing compensator that minimizes the harmful vibration of the vehicle body caused by road irregularities. This paper is organized as follows. In Chapter 2, the motivation background of this method for vehicle suspension design is summarized. In Chapter 3, the principle of the iterative scheme based on the mixed $H_2/H_\infty$ design is introduced. In Chapter 4, the steps of the iterative algorithm, i.e. the mixed $H_2/H_\infty$ controller design step, the LS identification step, and the verification step, will be presented with the whole algorithm. Finally, in Chapter 5, the applicability of the proposed algorithm for the active suspension design will be demonstrated.

2. Quarter-car model for suspension design

The well-known quarter-car vehicle model, which is shown in Figure 2.1, is widely used for active suspension design, because of its simplicity, low number of state variables and parameters, and because its easy to investigate the performance properties [11,19,20]. Let the vehicle body mass and axle mass be denoted by $m_b$ and $m_a$, the suspension stiffness and tire stiffness be denoted by $c_s$ and $c_t$. The quarter-car model contains two vertical degrees-of-freedom: let the displacement of the body mass and the axle mass be denoted by $y_b$ and $y_a$. In the quarter-car model, the disturbance, $d$, is caused by road irregularities. The input signal, $u$, is generated by the actuator controlled.

The motion equations of the quarter-car model can be formalized as follows:

$$
\begin{align*}
    m_b \ddot{y}_b + c_s y_b - c_s y_a + u &= 0, \\
    m_a \ddot{y}_a + c_s y_a + c_t y_a - c_s y_b - u - c_t d &= 0.
\end{align*}
$$

(2.1)

The road input is chosen to be Gaussian white-noise with autocorrelation function

$$
E[d(t)\bar{d}(t+\tau)] = S_v \delta(\tau),
$$

(2.2)

where $\delta(\tau)$ is the unit impulse and $S_v$ is a constant power spectral density, which depends on both road roughness and velocity displacement.

One of the most important difficulties in the suspension design is that the model contains a large number of components the behavior and properties of which are