19 Multi-Objective Control without Youla Parameterization

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Abstract. It is rather well-understood how to systematically design controllers that achieve multiple norm-bound specifications imposed on different channels of a control system. However, all known approaches to such problems are based on the Youla parametrization of all stabilizing controllers. This involves a transformation of the model description on the basis of a fixed stabilizing controller, and a wrong choice of this controller might require to unduly increase the controller order to closely approach optimality.

In this paper we suggest a novel procedure to multi-objective controller design which avoids the Youla parametrization and which directly applies to the generalized plant framework. In addition, we discuss various theoretical and practical numerical benefits of this new approach.

19.1 Introduction

In this paper we confine our attention to discrete-time linear time-invariant systems which admit a finite-dimensional state-space description. We use the standard notation to denote by $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ the input-output operator or the transfer matrix which is defined by the system $x(t+1) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + Du(t)$.

Consider a generalized plant with two performance channels and one control channel described as

$\begin{pmatrix} z_1 \\ z_2 \\ y \end{pmatrix} = \begin{bmatrix} A & B_1 & B_2 & B \\ C_1 & D_1 & D_{12} & E_1 \\ C_2 & D_{21} & D_2 & E_2 \\ C & F_1 & F_2 & 0 \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \\ u \end{pmatrix}.$ \hfill (19.1)

The inter-connection of (19.1) with a controller

$u = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} y = Ky \hfill (19.2)$

is denoted as

$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_1 & D_{12} \\ C_2 & D_{21} & D_2 \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{bmatrix} T_1(K) & T_{12}(K) \\ T_{21}(K) & T_2(K) \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}. \hfill (19.3)$
Problem Formulation. We consider the following multi-objective or structured $H_\infty$-control problem: Minimize $\gamma$ such that there exists a stabilizing controller (19.2) (a controller for which all eigenvalues of $A$ are located in the open unit disk) which renders the following $H_\infty$-norm constraints on the diagonal blocks of the controlled closed-loop system satisfied:

$$\|T_1(K)\|_\infty < \gamma \quad \text{and} \quad \|T_2(K)\|_\infty < \gamma.$$  \hspace{1cm} (19.4)

Let us denote the optimal achievable bound by $\gamma_*$.

Apart from being a mathematically challenging extension of standard single-objective $H_\infty$-control [3,5,7], our main practical motivation for such controller design techniques is as follows: They allow to enforce loop-shaping requirements with independent weights on unrelated subsystems of arbitrary closed-loop inter-connection without having to artificially include transfer matrix blocks of no interest [18].

In contrast to multi-objective control problems that involve different norm constraints on the diagonal blocks of the closed-loop system [1,2,19,13,6,15], this more specific problem has found attention in [12,11,10,14]. Without any exception, all existing approaches to approximately solve the genuine multiple norm controller design problems are based on the Youla parameterization. On the basis of a fixed stabilizing controller, one can determine a stable transfer matrix $T$ such that the set of all closed-loop transfer matrices $T(K)$ that result from stabilizing controllers $K$ is given by all

$$\begin{pmatrix} T_1 & T_{12} \\ T_{21} & T_2 \end{pmatrix} + \begin{pmatrix} T_{13} \\ T_{23} \end{pmatrix} Q \begin{pmatrix} T_{31} & T_{32} \end{pmatrix}$$  \hspace{1cm} (19.5)

if the Youla parameter $Q$ varies in the set $RH_\infty$ of all stable transfer matrices of appropriate dimension.

If (19.1) corresponds to a one-block problem ([4]) with $\begin{pmatrix} T_{13} \\ T_{23} \end{pmatrix}$ and $\begin{pmatrix} T_{21} & T_{31} \end{pmatrix}$ of full row and column rank on the whole unit circle respectively, rational interpolation theory allows to equivalently translate the multi-objective control problem into an LMI-problem [11,10]. This makes it possible to compute the optimal value and close-to-optimal controllers by solving a fixed-sized finite-dimensional optimization problem.

If the problem does not have a one-block nature, it has been suggested to perform a relaxation by designing a controller which minimizes an upper bound of the actual cost. The solution of these so-called mixed design problems [1,8,17,9] leads to controllers of the same order as the underlying plant, but it is generally hard to estimate in how far the upper bound relates to the exact optimal value.

All suggested solutions to solve the genuine multi-objective control problem proceed along the following lines. Choose a sequence of scalar stable transfer functions $q_0, q_1, \ldots$ which span a dense subspace in $RH_\infty$, where we