A Function State Space Approach to
Robust Tracking for Sampled-Data Systems

Yutaka Yamamoto

Abstract

It is well known that tracking to continuous-time signals by sampled-data systems presents various difficulties. For example, the usual discrete-time model is not suitable for describing intersample ripples. In order to adequately handle this problem we need a framework that explicitly contains the intersample behavior in the model. This paper presents an infinite-dimensional yet time-invariant discrete-time model which contains the full intersample behavior as information in the state. This makes it possible to clearly understand the intersample as a result of a mismatch between the intersample tracking signal and the system zero-directional vector. This leads to an internal model principle for sampled-data systems, and some nonclassical feature arising from the interplay of digital and continuous-time behavior.

1 Introduction

It is well known that tracking to continuous-time signals by sampled-data systems presents various difficulties. For example, the usual discrete-time model is not suitable for describing intersample ripples. In order to adequately handle this problem we need a framework that explicitly contains the intersample behavior in the model. There are now several approaches in this direction: [1], [2], [4], [12].

This paper employs the approach of [12], which gives an infinite-dimensional yet time-invariant discrete-time model, and this model contains the full intersample behavior as information in the state (a similar approach is also employed by [7] for computing $H^{\infty}$ norms). This makes it possible to clearly understand intersample ripples as a result of a mismatch between the intersample tracking signal and the system zero-directional vector. This leads to an internal model principle for sampled-data systems, and some nonclassical feature arising from the interplay of digital and continuous-time behavior.

2 Preliminaries

Let

$$\dot{z}(t) = Az(t) + Bu(t), \quad y(t) = Cz(t)$$  \hspace{1cm} (1)
be a given continuous-time system, and \( h \) a fixed sampling period. Given a function \( \psi(t) \) on \([0, \infty)\), we define \( \tilde{S}(\psi) \) as a sequence of functions on \([0, h)\) as follows:

\[
\tilde{S}(\psi) := \{ \psi_k(\theta) \}_{k=0}^{\infty}, \quad \psi_k(\theta) := \psi((k-1)h + \theta).
\] (2)

For such a sequence \( \{ \psi_k(\theta) \}_{k=1}^{\infty} \), its \( z \)-transform is defined as

\[
Z(\psi) = \sum_{k=0}^{\infty} \psi_k(\theta)z^{-k}.
\] (3)

We take \( \tilde{S}(u) = u_k(\theta), \tilde{S}(x) = x_k(\theta), \tilde{S}(y) = y_k(\theta) \) as input, state, and output vectors (although infinite-dimensional), and give a discrete-time transition rule. Indeed, at time \( t = kh \), let the state of the system \((A, B, C)\) be \( x_k(h) \) and let input \( u_{k+1}(\theta) \) \((0 \leq \theta < h)\) be applied to the system. Then the state and output trajectories \( x_{k+1}(\theta) \) and \( y_{k+1}(\theta) \) are given by the following equations:

\[
\begin{align*}
  x_{k+1}(\theta) &= e^{Ah}x_k(h) + \int_0^\theta e^{A(\theta-r)}Bu_{k+1}(\tau)d\tau, \\
  y_k(\theta) &= Cx_k(\theta), \quad 0 < \theta < h.
\end{align*}
\] (4)

Let \( U \) be the space of piecewise continuous functions on \((0, h]\) that are right-continuous. Introducing the operators

\[
\begin{align*}
  F : \ U^n &\to U^n : x(\theta) \mapsto e^{Ah}x(h), \\
  G : \ U^m &\to U^n : u(\theta) \mapsto \int_0^\theta e^{A(\theta-r)}Bu(r)dr, \\
  H : \ U^n &\to \nu^n : z(\theta) \mapsto Cz(\theta),
\end{align*}
\] (5)

equation (4) can be written simply as

\[
\begin{align*}
  x_{k+1} &= Fx_k + Gu_{k+1}, \\
  y_k &= Hx_k.
\end{align*}
\] (6)

Consider the hybrid control system depicted in Fig. 1. Here \( C(z) \) and \( P(s) \) denote discrete-time and continuous time systems \((A_d, B_d, C_d)\) and \((A_c, B_c, C_c)\), respectively. It is easy to