A Function State Space Approach to
Robust Tracking for Sampled-Data Systems

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Abstract

It is well known that tracking to continuous-time signals by sampled-data systems presents various difficulties. For example, the usual discrete-time model is not suitable for describing intersample ripples. In order to adequately handle this problem we need a framework that explicitly contains the intersample behavior in the model. This paper presents an infinite-dimensional yet time-invariant discrete-time model which contains the full intersample behavior as information in the state. This makes it possible to clearly understand the intersample as a result of a mismatch between the intersample tracking signal and the system zero-directional vector. This leads to an internal model principle for sampled-data systems, and some nonclassical feature arising from the interplay of digital and continuous-time behavior.

1 Introduction

It is well known that tracking to continuous-time signals by sampled-data systems presents various difficulties. For example, the usual discrete-time model is not suitable for describing intersample ripples. In order to adequately handle this problem we need a framework that explicitly contains the intersample behavior in the model. There are now several approaches in this direction: [1], [2], [4], [12].

This paper employs the approach of [12], which gives an infinite-dimensional yet time-invariant discrete-time model, and this model contains the full intersample behavior as information in the state (a similar approach is also employed by [7] for computing $H^\infty$-norms). This makes it possible to clearly understand intersample ripples as a result of a mismatch between the intersample tracking signal and the system zero-directional vector. This leads to an internal model principle for sampled-data systems, and some nonclassical feature arising from the interplay of digital and continuous-time behavior.

2 Preliminaries

Let

$$\dot{z}(t) = Az(t) + Bu(t), \quad y(t) = Cz(t)$$

(1)
be a given continuous-time system, and $h$ a fixed sampling period. Given a function $\psi(t)$ on $[0, \infty)$, we define $\hat{S}(\psi)$ as a sequence of functions on $[0, h)$ as follows:

$$\hat{S}(\psi) := \{\psi_k(\theta)\}_{k=0}^{\infty}, \quad \psi_k(\theta) := \psi((k - 1)h + \theta). \quad (2)$$

For such a sequence $\{\psi_k(\theta)\}_{k=1}^{\infty}$, its z-transform is defined as

$$Z(\psi) = \sum_{k=0}^{\infty} \psi_k(\theta) z^{-k}. \quad (3)$$

We take $\hat{S}(u) = u_k(\theta)$, $\hat{S}(x) = x_k(\theta)$, $\hat{S}(y) = y_k(\theta)$ as input, state, and output vectors (although infinite-dimensional), and give a discrete-time transition rule. Indeed, at time $t = kh$, let the state of the system $(A, B, C)$ be $x_k(h)$ and let input $u_{k+1}(\theta) (0 \leq \theta < h)$ be applied to the system. Then the state and output trajectories $x_{k+1}(\theta)$ and $y_{k+1}(\theta)$ are given by the following equations:

$$\begin{align*}
x_{k+1}(\theta) &= e^{A\theta} x_k(h) + \int_0^\theta e^{A(\theta - r)} Bu_{k+1}(r) dr, \\
y_k(\theta) &= Cx_k(\theta), \quad 0 < \theta < h. \quad (4)
\end{align*}$$

Let $U$ be the space of piecewise continuous functions on $(0, h]$ that are right-continuous. Introducing the operators

$$\begin{align*}
F : \quad &U^n \rightarrow U^n : x(\theta) \mapsto e^{A\theta} x(h), \\
G : \quad &U^m \rightarrow U^n : u(\theta) \mapsto \int_0^\theta e^{A(\theta - r)} Bu(\tau) d\tau, \\
H : \quad &U^n \rightarrow U^p : z(\theta) \mapsto Cz(\theta),
\end{align*}$$

equation (4) can be written simply as

$$x_{k+1} = Fx_k + Gu_{k+1}, \quad y_k = Hx_k. \quad (6)$$

Consider the hybrid control system depicted in Fig. 1. Here $C(z)$ and $P(s)$ denote discrete-time and continuous time systems $(A_d, B_d, C_d)$ and $(A_c, B_c, C_c)$, respectively. It is easy to

![Figure 1: Hybrid Closed-Loop System $\Sigma_{cd}$](image-url)