Robust controllers for infinite-dimensional systems*

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Abstract

This paper describes three theories for robust controller design for infinite-dimensional linear systems. They concern unstructured perturbations in the transfer function of additive, multiplicative and coprime-factor types. Explicit formulas are given for the maximal robustness margin, as well as for both infinite-dimensional and finite-dimensional robustly stabilizing controllers which achieve an a priori robustness margin.

1 Introduction

In section 3 of the previous article by Logemann [28], external closed-loop stability is defined for transfer matrices belonging to \( T^{p \times m} \), where \( T \) is a ring of fractions. This concept is closely related to the existence of coprime factorisations and in Theorem 16 of [28] a complete parametrization of all stabilizing controllers was obtained in terms of coprime factorizations of the transfer matrix and a free parameter, \( S \in S^{m \times p} \), the stable subset of \( T^{m \times p} \). This showed that there is an over abundance of controllers which will stabilize a system, but in practice, one requires a controller which does more than just that. For example, the nominal (or mathematical) model of a physical system is never exact and more often than not, it is a great simplification of reality. In other words, the nominal model is usually significantly different from the true system. Classical control designs like linear quadratic control designs produce a controller which is only guaranteed to stabilize the nominal model; it is not guaranteed to stabilize the true physical system. Clearly, a desirable property of the controller would be that it stabilizes not only the nominal plant, but all plants which are close to the nominal one in some sense. This type of property is called robust stabilization. There are many types of robustness one can consider, depending on the class of perturbations and the mathematical measure for distance one uses. In this paper we consider three specific types of robustness which allow for uncertainty in the transfer matrix and which measure distance in the \( L_\infty \)-norm. They are usually described as unstructured perturbations, as distinct from perturbations in the state-space operators, which are usually called structured perturbations. The attractive

*This paper was written during the author's sojourn at INRIA-Rocquencourt during January-June 1992.
feature of these three robustness problems is that they can all be formulated as $H^\infty$-optimal control problems, which were discussed in the article of Van Keulen [36] earlier in this volume. Nonetheless, we prefer not to use a state-space formulation, since these robustness problems can be solved more readily using a synthesis of state- and frequency-domain approaches. In particular, we use a coprime factorization description of the plant and our solutions are given in terms of linear fractional transformations of state-space realizations. These state-space realizations in turn depend on either Lyapunov or Riccati equations associated with a state-space realization of the original plant.

We formulate all three robust stabilization problems as $H^\infty$-optimal control problems and give solutions for two of these. In particular, we give formulas for the maximal robustness margin and explicit formulas for the robustly stabilizing controllers which achieve a certain robustness margin. In general, these are infinite-dimensional controllers, but we also give algorithms for designing finite-dimensional robustly stabilizing controllers with an a-priori robustness margin. Finally, we show that these robustly stabilizing controllers will not be destabilized by arbitrarily small delays.

2 Three robust stabilization problems

In this section, following the approach in MacFarlane and Glover [29], we formulate three different robust stabilization problems with respect to certain perturbations of the transfer function $G \in \mathbb{T}^{p \times m}$, where $\mathbb{T}$ is one of the quotient rings introduced in section 2 of Logemann [28]. In this paper, by stability we mean external stability as in Definition 7 of [28].

First, we introduce three classes of uncertainty models we shall consider.

Definition 2.1 Let $G$ and $G_\Delta$ be transfer matrices in $\mathbb{T}^{p \times m}$ of the nominal and the perturbed plants, respectively. Let $G = M^{-1}\hat{N}$ be a left-coprime factorization of $G$.

a. A perturbation $\Delta_A \in \mathbb{T}^{p \times m}$ is an additive uncertainty if

$$G_\Delta = G + \Delta_A$$

(2.1)

b. A perturbation $\Delta_p \in \mathbb{T}^{p \times p}$ is an multiplicative uncertainty if

$$G_\Delta = (I + \Delta_p)G$$

(2.2)

c. A perturbation $[\Delta_N, \Delta_M] \in \mathbb{S}^{p \times (m+p)}$ is a coprime-factor uncertainty if

$$G_\Delta = (\hat{M} + \Delta_M)^{-1}(\hat{N} + \Delta_N).$$

(2.3)

In order to prove some theorems about robust stabilization, we need to introduce some further restrictions on the class of plants $G$ and the class perturbations $\Delta$ allowed. $\mathcal{S}, \mathcal{T}$, $i = 1 - 4$ are defined as in section 2 of Logemann [28].