THREE METHODS FOR POSTOPTIMAL ANALYSIS IN INTEGER LINEAR PROGRAMMING

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Postoptimal analysis in LP is based on the existence of necessary and sufficient optimality conditions, for which complete information is contained in the optimal basic inverse. The same is not true for ILP. Optimality can only be proven through an algorithm. The possibility for postoptimal analysis is therefore dependent on the algorithm used to solve the ILP. In this paper we study methods for performing postoptimal analysis when the ILP has been solved by enumeration, branch-and-bound and cutting planes, respectively.

Key words: Integer Programming, Postoptimal Analysis, Cutting Planes, Additive Algorithm, Branch and Bound.

1. Introduction

Postoptimal analysis is fully understood and developed for linear programming (LP). When it comes to integer linear programming (ILP), little is known with respect to postoptimal analysis, and scattered results have only recently appeared in the literature (for a survey of these results see Geoffrion and Nauss [3]).

Approaches so far reported have been developed in the context of specific algorithms. It is therefore difficult to evaluate computational consequences of such methods. Moreover, there has been a tendency to extend outright the notion of sensitivity and parametric analysis from LP to ILP. Such analyses are closely related in LP to computational concepts, but no such concepts exist so far in ILP.

In this paper we survey three methods for postoptimal analysis in ILP. Each of them is based on a different algorithm for solving the ILP namely, enumeration, branch-and-bound and cutting planes.

In Section 2 we discuss postoptimal analysis for ILP in general and show that each algorithm consists in a sequence of tests, which determine sets of solutions, which cannot possibly be feasible nor optimal. These tests are called sufficiency tests, and it is information concerning these tests which is the basis for postoptimal analysis.

In Section 3 we survey three methods for postoptimal analysis, and in Section 4 the methods are illustrated on the same small numerical example.
2. Postoptimal analysis for linear and integer linear programming

In LP one distinguishes between different types of postoptimal analyses, such as sensitivity analysis, parametric analysis, addition of new variables or constraints, etc. Sensitivity analysis is concerned with the determination of ranges on problem parameters for which an optimal LP basis remains optimal; parametric analysis deals with the systematic exploration of simultaneous changes in problem parameters as the magnitude of these changes increases; etc. In all cases, the analysis begins with an optimal LP basis and rests on the utilization of LP optimality tests which are both necessary and sufficient. The sufficiency property is used to determine ranges on problem parameters for which the tests hold and the necessity property is used to guide the search to a new optimal solution when the tests become violated. These analyses are especially simplified in the case of LP, as the LP basis contains in compact form all information necessary to conduct the optimality tests.

One is interested in performing the same type of postoptimal analyses for ILP. However, although there does exist a duality theory for ILP (see Tind and Wolsey [13, 14, 15]) no simple necessary and sufficient optimality tests exist for ILP. Lack of simple optimality conditions necessitates the execution of an algorithm based on total or implicit total enumeration to prove optimality of any ILP solution. Any such algorithm, whether it is based on branch-and-bound, enumeration, cutting planes, etc., applies a sequence of sufficiency tests which determine sets of solutions which cannot possibly be feasible nor optimal and therefore can be eliminated from further consideration. Optimality is determined only after all but the clearly identified optimal solutions have been eliminated.

Postoptimal analysis implies changing problem parameters, and any change can invalidate any or all sufficiency tests previously employed in finding the optimal solution to the original problem. Postoptimal analysis in ILP, therefore, implies at least in principle, complete resolution of a new ILP problem. Most algorithms do allow for some limited postoptimal analysis, but it is based solely on the continued validity of the sufficiency tests. We call such methods and their resulting analyses trivial. (In this context, LP sensitivity analysis is a trivial analysis). Nontrivial postoptimal methods are those which allow for the determination of optimal solutions which lie outside the range of trivial analysis, i.e. a nontrivial method is able to determine the new optimal solution without complete resolution, even if a sufficiency test has been invalidated through parameter changes. All nontrivial methods for ILP postoptimal analysis differ from complete resolution in their use of information gained about the sufficiency tests while solving the original problem.

In the next section we survey three nontrivial methods for postoptimal analysis. The first one is based on solving the ILP by enumeration, the second on solving the ILP by branch-and-bound and the third is based on cutting planes.