On diffraction in a piezoelectric medium by a half-plane: The Sommerfeld problem

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Abstract. This paper is concerned with the diffraction problem in a transversely isotropic piezoelectric medium by a half-plane. The half-plane obstacle considered here is a semi-infinite slit, or a crack; both its surfaces are traction free and electric absorbent screens. In a generalized sense, we are dealing with the Sommerfeld problem in a piezoelectric medium.

The coupled diffraction fields between acoustic wave and electric wave are excited by both incident acoustic wave as well as incident electric wave; and the sound soft and electric “blackness” conditions on the screens are characterized by a system of simultaneous Wiener-Hopf equations. Closed form solutions are sought by employing special techniques. Some interesting results have been obtained, such as mode conversions between acoustic wave and electric wave, novel diffraction patterns in the scattering fields, and the effect of electroacoustic head wave, as well as of surface wave—Bleustein-Gulyaev wave.

Unlike the classical Sommerfeld problem, in which the only concern is the scattering field of electric wave, the strength of material, e.g. material toughness, is another concern here. From this perspective, relevant dynamic field intensity factors at the crack tip are derived explicitly.

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1. Introduction

The study of wave scattering problems in piezoelectric media is of particular importance, because many piezoelectric devices are exclusively made as wave guides, which can either enhance acoustic wave, or transfer acoustic energy to electric energy, and vice versa for practical purposes. In fact, wave propagation in the piezoelectric medium is a unique embodiment of acoustic wave and electromagnetic wave, both of which are the paradigms of linear hyperbolic systems of partial differential equations, which attributes special significance to such study. Unfortunately, if not surprisingly, it appears to this author that there is a lack of fundamental understanding on the subject. This work attempts to provide a systematic analysis on a half-plane scattering problem in a transversely isotropic piezoelec-
tric medium, which is generated by both shear-horizontal (SH) acoustic wave and transverse electric (TE) wave.

There is a dilemma in studying scattering problems in piezoelectric media, even if one is only interested in electroacoustic wave. In general, the fully coupled Christoffel-Maxwell or Euler-Maxwell equations are hardly tractable. In order to simplify the problem, quasi-static approximation is widely adopted. The setback of quasi-static approximation is that it leads to the loss of hyperbolicity of the simplified system, and subsequently prevent any meaningful analysis on transient problems in piezoelectric medium. Almost all of the previous attempts on the subject were made within the realm of quasi-static approximation, consequently, the results obtained are numeric in nature (e.g. Auld [1973b], Parton & Kudryatvsev [1988], and Shindo et al. [1990]). To improve the situation, Li [1998] proposed a so-called “quasi-hyperbolic approximation” for a class of transversely isotropic piezoelectric media, and the purpose of this “quasi-hyperbolic approximation” is to preserve the hyperbolicity of the simplified system, and at the same time the simplified system can still enjoy the simplicity that the “quasi-static” approximation provided before.

Consider a traction free and perfectly conducting crack (with its both surfaces as absorbent screens), which is located at the positive part of $x_1$ axis, namely,

$$
\begin{align*}
\sigma_{23}(x_1,0,t) &= \sigma_{23}^{(s)}(x_1,0,t) + \sigma_{23}^{(i)}(x_1,0,t) = 0, \quad (a) \\
\phi(x_1,0,t) &= \phi^{(s)}(x_1,0,t) + \phi^{(i)}(x_1,0,t) = 0, \quad (b)
\end{align*}
$$

(1.1)

The condition (1.1(b)) (absorbent screen) renders it as a Sommerfeld problem (Sommerfeld [1896], [1901], and [1949]) in a generalized sense. From the point of view of mathematical physics, this is a mixed Dirichlet-Neumann, or Robin problem; the interplay between mechanical field and electric field along the boundary generates both symmetric and anti-symmetric scattering fields. Subsequently, there may exist a nonzero bulk scattering displacement field as well as non-zero electrical potential field at the crack tip, which is fundamentally different from the conventional diffraction problems by cracks (there are some good examples in Sih [1977]).

2. Problem statement

Based on the “quasi-hyperbolic approximation” (see Li [1998]), for hexagonal symmetry piezoelectric materials (e.g. 6mm class), the relevant electromechanical

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†In this paper, the term electro-acoustic wave is reserved for electric potential disturbance travelling at sound speed; whereas the term electroacoustic wave is referred to as both acoustic wave as well as electro-acoustic wave.