Stochastic evolution with slow learning*

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Summary. This paper studies the extent to which diffusion approximations provide a reliable guide to equilibrium selection results in finite games. It is shown that they do for a class of finite games with weak learning provided that limits are taken in a certain order. The paper also shows that making mutation rates small does not in general select a unique equilibrium but making selection strong does.

Keywords and Phrases: Equilibrium selection, Diffusion approximation, Evolutionary game theory, Risk dominance.

JEL Classification Numbers: C73.

1 Introduction

This paper considers stochastic models of equilibrium selection in games. There has been substantial interest in evolutionary models of equilibrium selection since the work of Kandori, Mailath and Rob (1993) and Young (1993). This work has explored the idea that models of evolution with random shocks may help predict which equilibrium will be played in games with multiple equilibria. Most of this work has considered models of selection in discrete time. Models of equilibrium selection in continuous time (see for example Foster and Young, 1990; Fudenberg and Harris, 1992) have sometimes given rather different answers and it is therefore of some interest to understand the relationship between the two approaches.

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The typical approach in the papers inspired by Kandori, Mailath and Rob (1993) and Young (1993) is to take some model of dynamic adjustment and perturb it (say by introducing mutations) so that the resulting Markov process has a unique stationary distribution. The perturbation is then allowed to tend to zero and the question is which of the equilibria of the unperturbed process does the stationary distribution converge to. This equilibrium is then taken to be the one selected by evolution. A notable result of Kandori, Mailath and Rob is that the equilibrium selected in symmetric $2 \times 2$ co-ordination games is the risk-dominant equilibrium in the sense of Harsanyi and Selten (1988), under minimal assumptions on the underlying deterministic dynamic. Ellison (1995) generalises this result. Bergin and Lipman (1996) discuss its limitations.

The work above considers games with a fixed finite population. Many commonly studied dynamics (for example the replicator dynamic) are often studied using differential equations, that is with a continuum of players. Foster and Young (1990) and Fudenberg and Harris (1992) consider perturbations of differential equations by small stochastic noise. In their context, the equilibrium selected depends rather closely on the chosen dynamic. This contrast with the finite models is perhaps disturbing. This paper aims to investigate the connection between finite population and differential equation models.

Some previous papers have considered finite population models when the number of players is large. If the source of randomness in the model is disturbances at the individual level and the strength of selection remains fixed regardless of sample size, then, by the law of large numbers, one would expect uncertainty to average out at the aggregate level. The sample paths for a large population should therefore be well approximated by the solutions to a deterministic differential equation. This is indeed the case if one considers behaviour over a fixed finite interval of time. Binmore, Samuelson and Vaughan (1994) and Sandholm (1999), for example, provide proofs in an economic context. Ethier and Kurtz (1986, Chapter 11) have a general treatment. Nevertheless this approximation may not be helpful if one is interested in the stationary distribution, which involves an indefinite time interval. In particular, the finite process may have a unique stationary distribution for each population size but the differential equation may have multiple stationary points.

This paper pursues a different approach. In the papers considered above, the learning process at the aggregate level is almost deterministic when the population size is large. For some models the force of selection or learning may be weak or slow at the individual level, even though it is important in the aggregate. In these cases there may be significant randomness even in a large population. To capture this, this paper considers taking the limit where the strength of selection at the individual level becomes small as the population size becomes large. The force of this assumption is not that this is literally so but that selection is weak at the individual level in comparison to the size of the population. One might draw a loose analogy with the Poisson approximation to the Binomial, which is applied when the Binomial probabilities are small in comparison to the sample size.