Approximating Latin Square Extensions

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Abstract. In this paper we investigate the problem of computing the maximum number of entries which can be added to a partially filled latin square. The decision version of this question is known to be NP-complete. We present two approximation algorithms for the optimization version of this question. We first prove that the greedy algorithm achieves a factor of $\frac{1}{3}$. We then use insights derived from the linear relaxation of an integer program to obtain an algorithm based on matchings that achieves a better performance guarantee of $\frac{1}{2}$. These are the first known polynomial-time approximation algorithms for the latin square completion problem that achieve nontrivial worst-case performance guarantees. Our study is motivated by applications to lightpath assignment and switch configuration in wavelength routed multihop optical networks.

Key Words. Latin squares, Optical switch configuration, Approximation algorithms.

1. Motivation

1.1. Optical Networks. Developments in fiber-optic networking technology using wavelength division multiplexing (WDM) have finally reached the point where it is considered the most promising candidate for the next generation of wide-area backbone networks. These are highly flexible networks capable of supporting tens of thousands of users and capable of providing capacities on the order of gigabits-per-second per user [CNW], [Gr], [Ram]. WDM optical networks utilize the large bandwidth available in optical fibers by partitioning it into several channels each at a different optical wavelength [BH1], [CNW], [IEE], [IK].

A typical optical network consists of routing nodes interconnected by point-to-point fiber-optic links. Each link supports a certain number of wavelengths. The routing nodes are capable of photonic switching, also known as dynamic wavelength routing which involves the setting up of lightpaths [CB], [CGK], [ZA]. A lightpath is an optical path between two nodes on a specific wavelength. The optical switch at a node assigns the wavelengths from an incoming port to an outgoing port. This assignment is alterable and can be controlled electronically.

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Conflict-free wavelength routing in wide-area optical networks is achieved by utilizing \textit{latin routers} [BH2]. These are routing devices that employ the concept of a \textit{latin square} (LS). A latin router with $n$ input ports, $n$ output ports, and $n$ wavelengths is associated with a \textit{partial latin square} (PLS), an $n \times n$ matrix that specifies the wavelength connections from the $n$ input ports to the $n$ output ports. The matrix contains elements from the set $\{0\} \cup \{1, 2, \ldots, n\}$ ($0$ is used as a placeholder to denote emptiness) such that each row and each column never contains an element from the set $\{1, 2, \ldots, n\}$ more than once (see Figure 1 for an example). A nonzero entry $L_{ij}$ of $L$ means that the wavelength $L_{ij}$ is routed from input port $i$ to output port $j$. A zero entry denotes an unassigned entry. An LS is a PLS that has no zero entries.

Reducing the number of unassigned or zero entries in the PLS associated with a router is of paramount practical importance in optical networks as this ensures reduced wastage of the valuable resources of ports and wavelengths. This motivates the following definitions:

\textbf{Definition 1.} A PLS $S_1$ is said to extend or be an extension of a PLS $S_2$ if $S_1$ can be obtained by altering only zero entries of $S_2$.

\textbf{Definition 2.} A PLS is said to be completable if it can be extended to an LS.

See Figure 1 for an LS obtained by extending the PLS of Figure 1. Not all PLSs can be completed (see Figure 2).

\textbf{Definition 3 (Partial Latin Square Extension Problem (PLSE)).} Given a PLS $S_1$ find the largest number of zero entries that can be changed to obtain a PLS $S_2$ that is an extension of $S_1$.

The PLSE problem as stated above is an optimization problem. The natural decision version of the problem—namely, given a PLS establish whether it is completable—has been shown to be \textit{NP}-complete [Co]. We present the first known polynomial-time approximation algorithms for the PLSE problem with nontrivial worst-case performance guarantees.

1.2. \textit{Other Applications}. This study also has applications to the more classical areas of statistical designs and error-correcting codes. We refer the interested reader to the (extensive) literature on the subject [DK1], [DK2].