Portfolio optimization problem under concave transaction costs and minimal transaction unit constraints

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Abstract. We will propose a branch and bound algorithm for calculating a globally optimal solution of a portfolio construction/rebalancing problem under concave transaction costs and minimal transaction unit constraints. We will employ the absolute deviation of the rate of return of the portfolio as the measure of risk and solve linear programming subproblems by introducing (piecewise) linear underestimating function for concave transaction cost functions. It will be shown by a series of numerical experiments that the algorithm can solve the problem of practical size in an efficient manner.

Key words. portfolio optimization – concave transaction cost – rebalancing – minimal transaction unit – branch and bound algorithm – global optimization

1. Introduction

This paper is concerned with a portfolio optimization problem under nonconvex (concave) transaction costs and minimal transaction unit (MTU) constraints.

An investor has to pay a certain amount of fees when he/she purchases (invest) and/or sells (disinvest) assets. Let $X_j$ be the amount of investment (or disinvestment) of the $j$th asset ($j = 1, \cdots, n$). The transaction cost associated with $X = (X_1, \cdots, X_n)$ is usually defined as the sum of the functions $\sum_{j=1}^{n} C_j(X_j)$ where each function $C_j(X_j)$ is a non-decreasing concave function up to certain point $\bar{X}_j$ as shown in Fig. 1. This is because the unit transaction cost is relatively large when $X_j$ is small and it gradually decreases as $X_j$ increases. However, the unit transaction cost increases beyond some point $\bar{X}_j$, due to the “illiquidity” effects. Let us briefly explain this phenomena.

Investment (disinvestment) of assets of an investor is associated with the disinvestment (investment) of other investors. If $X_j$ is large and if there is not enough supply (demand), then the unit price will increase (decrease), so that $C_j(X_j)$ becomes convex beyond point $A_j$.

We will assume in this paper that the amount of investment (disinvestment) is below the critical point, where the transaction cost is a well specified concave function.
calculated by the transaction cost table of the agent. The portfolio optimization problem to be treated in this paper becomes the minimization of a separable concave function under linear constraints. This type of problems are abundant in the literature of operation research. However, only a few problems have been solved to global optimality, at least until recently.

People often employ a linear (convex) approximation of a transaction cost function, or entirely ignore it and adjust the resulting solution in a heuristic manner. This manipulation may be adequate under certain circumstances, particularly if the amount of transaction is large when a linear approximation or heuristic approach may lead to a good solution. However, this approach may produce an erroneous result when the amount of investment is small and it is allocated to many assets in smaller fractions. Alternatively, one may apply integer programming approach by using piecewise linear approximation of nonconvex cost functions. However, as reported in [7], it takes a huge amount of computation time.

Fortunately, due to the recent progress in global optimization, one can now solve a fairly large scale linearly constrained concave minimization problem using the special structure of the problem (See [5,8,17] for such examples). This paper adds to the list of these successful examples.

The first serious effort in portfolio optimization under nonconvex cost structure was undertaken by Perold [15] in 1984. He proposed a piecewise linear convex approximation of the cost function and implemented it in the highly reputed software “Optimizer”. However, it is argued among the users of this software that this transaction cost routine requires a significant amount of computation time.

More recently, authors proposed a branch and bound algorithm for solving a concave cost portfolio optimization problem under the mean-absolute deviation framework. We used a linear underestimating function for the concave cost function and solved the resulting linear subproblems in a branch and bound using a well-designed problem reduction technique. We showed in [10] that this algorithm generates a good solution in a very efficient manner. We will show in this paper that the solution obtained by this