Markov-functional interest rate models

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Abstract. We introduce a general class of interest rate models in which the value of pure discount bonds can be expressed as a functional of some (low-dimensional) Markov process. At the abstract level this class includes all current models of practical importance. By specifying these models in Markov-functional form, we obtain a specification which is efficient to implement. An additional advantage of Markov-functional models is the fact that the specification of the model can be such that the forward rate distribution implied by market option prices can be fitted exactly, which makes these models particularly suited for derivatives pricing. We give examples of Markov-functional models that are fitted to market prices of caps/floors and swaptions.

Key words: Yield curve modelling, derivatives pricing, Markov processes

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1 Introduction

Amongst practitioners in the interest rate derivatives market a consensus is starting to emerge as to the desirable and most important properties of an interest rate pricing model. These properties stem from the way these models are used in practice. To determine the prices of exotic derivatives, pricing models are used as ‘extrapolation tools’. First the model parameters are chosen so that the model values of relevant liquid instruments agree with market prices, then the model is...
used to price the exotic. From this perspective, the properties of a good pricing model for derivatives can be summarised as follows. The model should

a) be arbitrage-free;
b) be well-calibrated, correctly pricing as many relevant liquid instruments as possible without overfitting;
c) be realistic and transparent in its properties;
d) allow an efficient implementation.

So far, models proposed in the literature have not been able to combine all four requirements. The contribution of this paper is to present a modelling framework within which one can develop models possessing all four attributes.

The *conditio sine qua non* for all models is, of course, freedom from arbitrage. The general framework for arbitrage-free interest rate models was laid out by Heath et al. (1992), HJM hereafter. All models we will consider are arbitrage-free interest rate models and representatives of the class of HJM models.

The traditional approach for specifying an interest rate model is to take one or more mathematically convenient underlying variables and to make certain distributional assumptions to reflect the future uncertainty of these variables. One possible choice for the underlying variables are the instantaneous forward rates. This is the setup for the model proposed by HJM. Unfortunately, for most distributional assumptions one can make for the forward rates (in the HJM framework this is known as ‘specifying the volatility function’), the dynamics of the forward rates become path-dependent which makes numerical implementations of the model very cumbersome. An interesting sub-class was proposed independently by Cheyette (1992) and Ritchken and Sankarasubramanian (1995). They find restrictions on the volatility functions such that the path-dependency can be reflected by one additional state-variable, which makes reasonably efficient numerical implementation possible. Instead of explicitly restricting the volatility functions one can also choose not to model the complete forward curve directly, but to focus attention on a single rate, the instantaneous short rate. A few of the best known examples of this approach are the models proposed by Hull and White (1994), Cox et al. (1985) or Black et al. (1990) and Black and Karasinski (1991). These models assume that the short rate has a normal, chi-square or log-normal distribution respectively. The attractive feature of these models is their simple and efficient numerical implementation.

A general problem with models from the traditional approach is that neither the instantaneous forward rates, nor the instantaneous short rate can be observed in the market. To make matters worse, prices for instruments which can be observed in the market such as caps, floors and swaptions often depend in complicated non-linear ways on the underlying model parameters. Hence, to replicate market prices one chooses a set of reference instruments and solves for the ‘correct’ parameter values by solving a non-linear optimization problem such that the pricing error of the model is minimised. These procedures can be numerically quite intensive and are known to be plagued by numerical instabilities.