New results in $\rho^0$ meson physics

M. Benayoun\textsuperscript{1}, S. Eidelman\textsuperscript{2}, K. Maltman\textsuperscript{3,4,5}, H.B. O’Connell\textsuperscript{4,6}, B. Shwartz\textsuperscript{2}, A.G. Williams\textsuperscript{4,5}

\textsuperscript{1} LPNHE des Universités Paris VI et VII-IN2P3, Paris, France (e-mail: benayoun@in2p3.fr)
\textsuperscript{2} Budker Institute of Nuclear Physics, Novosibirsk 630090, Russia (e-mail: eidelman@vxcern.cern.ch)
\textsuperscript{3} Mathematics and Statistics, York University, 4700 Keele St., North York, Ontario, Canada M3J 1P3\textsuperscript{a}
(e-mail: maltman@fewbody.phys.yorku.ca)
\textsuperscript{4} Department of Physics and Mathematical Physics, University of Adelaide 5005, Australia
(e-mail: awilliam@physics.adelaide.edu.au)
\textsuperscript{5} Special Research Centre for the Subatomic Structure of Matter, University of Adelaide 5005, Australia
\textsuperscript{6} Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA\textsuperscript{b}
(e-mail: hoconnel@ruthless.pa.uky.edu)

Received: 1 August 1997 / Revised version: 4 December 1997 / Published online: 26 February 1998

Abstract. We compare the predictions of a range of existing models based on the Vector Meson Dominance hypothesis with data on $e^+e^- \rightarrow \pi^+\pi^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ cross-sections and the phase and near-threshold behavior of the timelike pion form factor, with the aim of determining which (if any) of these models is capable of providing an accurate representation of the full range of experimental data. We find that, of the models considered, only that proposed by Bando et al. is able to consistently account for all information, provided one allows its parameter $a$ to vary from the usual value of 2 to 2.4. Our fit with this model gives a point–like coupling $\gamma\pi\pi$ of magnitude $\simeq -e/6$, while the common formulation of VMD excludes such a term. The resulting values for the $\rho$ mass and $\pi^+\pi^-$ and $e^+e^-$ partial widths as well as the branching ratio for the decay $\omega \rightarrow \pi^+\pi^-$ obtained within the context of this model are consistent with previous results.

1 Introduction

Our aim is to study the various ways to describe the $\rho^0$ meson in order to find an optimum modelling able to account most precisely for the known features of the physics involving this meson. This is mainly motivated by the fact that a precise knowledge of its properties is of fundamental importance in several fields of particle physics. It is important to emphasise that our philosophy is to look for the simplest models as these are the most useful in application to other systems, due to their ease of implementation. Naturally, such models should, as much as possible, respect basic general principles such as gauge invariance and unitarity. One must keep in mind that any parameters quoted for a given model are relevant only to that model. Indeed, a study of the model-dependence of resonance parameters is one of the principal goals of this work. It should also be noted that while each of these models is related to some underlying effective field theory through an effective Lagrangian, the models we are using here are simple amplitudes arising from an assumption of almost complete $s$-channel resonance saturation. The appropriateness of this assumption away from the resonance region can only be judged by quantitative studies of higher order (e.g., loop) effects in the corresponding effective field theories. While this is clearly a very important task, it is not our concern here and will not be considered further.

For this purpose, we study the strong interaction corrections to one-photon mediated processes in the low energy region where QCD is non-perturbative. To do this we shall look at two related processes, $e^+e^- \rightarrow \pi^+\pi^-$ and $e^+e^- \rightarrow \mu^+\mu^-$. The effect of the strong interaction is obvious in the first reaction and provides a large enhancement to the production of pions in the vector meson resonance region [1–4]. This enhancement, relative to what would be expected for a structureless, pointlike pion, is reflected in the deviation of the pion form factor, $F_\pi(q^2)$, from 1, and is primarily associated with the $\rho$ meson (where $q_\mu$ is the four momentum of the virtual photon). This form factor is successfully modelled in the intermediate energy region using the vector meson dominance (VMD) model [5]. VMD assumes that the photon interacts with physical hadrons through vector mesons and it is these mesons that give rise to the enhancement, through their resonant (possessing a complex pole) propagators of the form

\[ D_{\mu\nu}(q^2) = \frac{-g_{\mu\nu}}{q^2 - m_V^2 + im_V\Gamma_V(q^2)}, \] (1)

where $m_V$ and $\Gamma_V$ are the (real valued) mass and the momentum-dependent width. (Here we have included only
Traditionally, VMD assumes that all photon–hadron coupling is mediated by vector mesons. However, from an empirical point of view, one has the freedom, motivated by Chiral Perturbation Theory (ChPT) to include other contributions to such interactions. For instance, in a fit to $F_\pi(q^2)$ [6], it was shown that a non-resonant photon–hadron coupling can be accommodated merely by shifting the mass and width of the $\rho^0$ by about 10 MeV. Thus, the values extracted from $F_\pi(q^2)$ for the $\rho$ mass, $m_\rho$, and width, $I_\rho$, are model-dependent and in quoting values for them, the model used should be clearly stated. We note in passing that to define vector meson masses and widths in a process-independent way, one should refer to the location of the corresponding complex pole in the S-matrix. One should, however, bear in mind that alternate, and more traditional definitions of the mass and width not tied to the location of the S-matrix pole (for example a definition of $m_\rho^2$ as that value of $q^2$ for which the $P$-wave $\pi\pi$ phase shift passes through $90^\circ$) are in general specific to the process employed in the definition. The process-dependence of such alternate mass and width definitions has in fact led a number of authors to advocate using the S-matrix pole position to provide a process-independent definition of the $Z^0$ mass and width in the Standard Model [7].

Naturally, VMD can be applied to many other systems. We can consider the process $\eta'/\eta \rightarrow \pi^+\pi^-\gamma$ taking place through a combination of resonant (such as $\eta'/\eta \rightarrow \rho\gamma \rightarrow \pi^+\pi^-\gamma$) and non-resonant channels. In this manner an acceptable fit to data can be achieved with a range of combinations for the $\rho$ parameters and non-resonant terms [6,8]. Recent interest in this process has centred on the non-resonant term, which, if it arises from anomalous box and triangle diagrams, provides a possible test of QCD [9–11]. However, to determine the size of any non-resonant contribution, the resonant meson parameters need to be well fixed [6,8], and thus $F_\pi(q^2)$ well understood. This is one of the main aims of this paper.

We thus turn our attention to $e^+e^- \rightarrow \mu^+\mu^-$. In modelling the strong interaction correction to the photon propagator, VMD assumes that the strong interaction contribution is saturated by the spectrum of vector meson resonances [12]. Therefore, in principle, we can extract information on the vector meson parameters (independently of the $e^+e^- \rightarrow \pi^+\pi^-$ fit) without having to worry about non-resonant processes. However, as the vector mesons enter in $e^+e^- \rightarrow \mu^+\mu^-$ with an extra factor of $\alpha$ compared with $e^+e^- \rightarrow \pi^+\pi^-$, their contributions are considerably suppressed, making their extraction difficult. For this reason, we shall perform a simultaneous fit to both sets of data, in order to impose the best possible constraint on the vector meson parameters, and see if existing muon data are already precise enough in order to constrain the $\rho^0$ parametrisation.

Another way to constrain the descriptions of the $\rho^0$ meson is to compare the strong interaction $\pi\pi$ phase obtained using the various VMD parametrisations determined in fitting $e^+e^- \rightarrow \pi^+\pi^-$ with the corresponding phase [13] obtained using $\pi\pi$ scattering data and the general principles of quantum field theory, as well as the near–threshold predictions of ChPT. This happens to be more fruitful and conclusive in showing how VMD should be dealt with in order to reach an agreement with a large set of data and with the basic principles of quantum field theory.

The hadronic dressing of the photon propagator (the one-photon irreducible self-energy $\Pi^{\text{had}}(q^2)$) is also of interest for another reason. The anomalous magnetic moment of the muon can now be measured to such accuracy [14] that the strong interaction correction is important [15]. This needs to be completely understood if one is to look for physics beyond the Standard Model in this quantity. At present the correction is inferred from $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ using dispersion theory and the optical theorem or estimated with hadronic models (which, being non-perturbative, are difficult to use). The process $e^+e^- \rightarrow \mu^+\mu^-$ allows for a direct examination of the strong interaction modification to the photon propagator. Ideally we would obtain the low energy corrections to the photon propagator experimentally, and be able to use QCD perturbatively for higher energies (though the threshold above which we can ignore non-perturbative effects is difficult to determine [16]). The $\rho$ meson has already been seen in $e^+e^- \rightarrow \mu^+\mu^-$ [1,4]. It is only noticeable around the pole region where, due to its small width (4.4 MeV), it produces a sharp peak easily seen in the available data. The large width of the $\rho$ further suppresses the $\rho$ and $\omega$ contributions.

The outline of the present work is as follows. In Sect. 2 we describe the various formulations of the VMD assumption and the unitarisation procedure; we also discuss the phase definition relevant for our purpose. The fit procedure is sketched in Sect. 3 and implemented in Sect. 4. Comparison with the isospin 1 $P$–wave $\pi\pi$ phase shift deduced from $\pi\pi$ scattering theory is the subject of Sect. 5, while in Sect. 6 we give the near–threshold parameter values that are deduced from our fits of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section and they are compared to predictions and to other experimental determinations. The results obtained are discussed in Sect. 7 where we present our optimal fit for the $\rho^0$ parameters and the model which accounts for its properties the most appropriately. Finally, we summarise our conclusions in Sect. 8.

2 Vector meson models

We shall now provide a description of the various models we will use to fit the data for both $e^+e^- \rightarrow \pi^+\pi^-$ and $e^+e^- \rightarrow \mu^+\mu^-$. The cross-section for $e^+e^- \rightarrow \pi^+\pi^-$ is given by (neglecting the electron mass)

$$\sigma = \frac{\pi \alpha^2 (q^2 - 4m_e^2)^{3/2}}{3 (q^2)^{5/2}} |F_\pi(q^2)|^2,$$

where the form factor, $F_\pi(q^2)$ is determined by the specific model. Similarly, $F_\mu(q^2)$ is defined to be the form factor for the muon, and the cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ is