Deep inelastic scattering with a tagged photon: QED corrections for the $\Sigma$ method

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Abstract. After a brief review of the kinematics of deep inelastic scattering (DIS) within the so-called $\Sigma$ method, we derive the necessary formulae for the treatment of QED radiative corrections to DIS originating from hard photon radiation. The results are applied to a calculation of the corrections to DIS with a tagged photon with next-to-leading logarithmic accuracy under HERA conditions. It turns out that the next-to-leading logarithmic corrections are quite important for the $\Sigma$ method. We also discuss the dependence of the corrections on the longitudinal structure function of the proton, $F_L$, in the region of low $Q^2$ and moderate $x$.

1 Introduction

The determination of the structure functions of the proton, $F_2(x,Q^2)$ and $F_L(x,Q^2)$, over a broad range of the kinematic variables belongs to the most important tasks of the H1 and ZEUS experiments at the HERA $ep$ collider. Especially the extension of these measurements to the range of small Bjorken $x < 10^{-4}$ and $Q^2$ of a few GeV$^2$ is of particular interest, as it provides a testing ground for our attempts to understand the details of the dynamics of quarks and gluons inside the nucleon.

Whilst the structure function $F_2$ can be extracted quite easily from the experimental data, it is more difficult to determine the longitudinal structure function $F_L$. A direct method that relies only on measured data requires running the collider at different center-of-mass energies. However, besides impairing the high-energy program of the machine, running at reduced beam energies also increases some systematic errors, (e.g., luminosity uncertainties), in the experimental analysis.

These problems are circumvented by employing a method suggested by Krasny et al. [1] that utilizes radiative events. This method takes advantage of a photon detector (PD) in the very forward direction, as seen from the incoming lepton (electron or positron) beam. Such a device is part of the luminosity monitoring system of both the H1 and ZEUS experiments.

The idea of this method is that emission of photons in a direction close to the incoming lepton corresponds to a reduction of the effective beam energy. This effective beam energy for each radiative event is determined from the energy of the hard photon observed (tagged) in the PD. Early analyses that make use of these radiative events for a determination of $F_2$ were already published in [2,3]. No QED radiative corrections were taken into account in these analyses. The feasibility of a determination of $F_L$ was studied in [4].

Recently, the H1 collaboration presented preliminary results of a refined analysis with newer data [5]. In this analysis, the authors chose different methods of determination of the kinematic variables$^1$ (the $e$-method, where the kinematic variables are obtained from a measurement of the scattered lepton, and the $\Sigma$ method) in different ($x,Q^2$) bins in order to reduce the experimental systematic error. However, since the calculations of the QED radiative corrections to DIS with a tagged photon [7-9] did not cover the $\Sigma$ method, the corrections were only applied to part of the data in [5]. It is the purpose of the present work to extend these analytical calculations to the $\Sigma$ method.

The $\Sigma$ method, as proposed by Bassler and Bernardi [10], tries to combine the momenta of the outgoing lepton and hadrons judiciously in order to reduce experimental systematic uncertainties on the determination of the kinematic variables especially in the kinematic region of low $Q^2$ where other methods are limited by e.g., detector resolution or energy calibration.

With the help of the quantity$^2$
\begin{equation}
\Sigma_h \equiv \sum_h (E_h + p_{z,h}) ,
\end{equation}

$^1$ For a discussion of the most common methods to determine kinematic variables and further references, see e.g. [6]

$^2$ Note that in this paper we take the positive $z$-axis along the initial lepton direction, unlike [10] who chose the direction of the incoming proton beam
where the sum runs over the detected hadrons, and \( E_h \) and \( p_{z,h} \) are the energy and \( z \)-component of the respective particle, the kinematic variables \( x_\Sigma \), \( y_\Sigma \) and \( Q^2_\Sigma \) are defined via

\[
y_\Sigma = \frac{\Sigma_h + E'_e (1 + \cos \theta)}{1 - y_\Sigma}, \quad Q^2_\Sigma = \frac{E^2_e \sin^2 \theta}{1 - y_\Sigma}, \quad x_\Sigma = \frac{Q^2_\Sigma}{y_\Sigma S}.
\]

Here \( S = 4E_e E_p \), where \( E_e \) and \( E_p \) are the beam energies of the lepton and proton beam, respectively, and \( E'_e \) and \( \theta \) are the energy and scattering angle of the outgoing lepton, measured with respect to the direction of the initial lepton.

One of the known advantages of the \( \Sigma \) method is its insensitivity of the determination of \( y_\Sigma \) and \( Q^2_\Sigma \) to undetected emission of a hard photon collinear to the incoming lepton (initial state radiation, ISR).

The \( \Sigma \) method has already been used in several analyses of the H1 collaboration. However, the author is not aware of any publications on an analytical (i.e., non-Monte Carlo) treatment of QED radiative corrections to deep inelastic scattering (DIS) using the \( \Sigma \) kinematic variables\(^3\) beyond the collinear (leading log) approximation \([12, 13]\). The presumable reason is that the \( \Sigma \) method has only been introduced long after the start of data taking at HERA. Therefore, this paper starts with a brief introduction to the kinematics of radiative DIS in the \( \Sigma \) method. Section 3 extends the considerations to the case of DIS with an exclusive tagged photon, but specialized to the conditions at HERA, and provides the relevant formulae to calculate the radiative corrections to the tagged photon cross section, based on the results of \([9]\). Some results for HERA experimental conditions are presented in Sect. 4, and Sect. 5 contains our conclusions. Finally, the appendices collect several technical details.

### 2 Kinematics in the \( \Sigma \) method

This section is devoted to a basic review of the \( \Sigma \) method. Here we shall prepare an appropriate framework for the treatment of radiative corrections to radiative deep inelastic scattering,

\[
e(p) + P(P) \to e(p') + X(P') + \gamma(k),
\]

i.e., DIS with single hard photon emission. The extension to the process with an additional tagged photon in the forward direction is straightforward and will be performed in the next section.

Let us begin by stating our conventions for the kinematics in the HERA lab frame that are used throughout this paper. We shall take the orientation of the coordinate frame such that the positive \( z \)-axis points in the direction of the incoming lepton beam, and the momentum of the scattered lepton lies in the \( x,z \)-plane:

\[
P = (E_p, 0, 0, -p_p), \quad p = (E_e, 0, 0, p_e), \quad p' = (E'_e, p'_e \sin \theta, 0, p'_e \cos \theta), \quad k = E_e (1, \sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \vartheta).
\]

As the beam energies \( E_p, E_e \), as well as the energy of the scattered lepton, \( E'_e \), are always large compared to the proton mass, \( M \), and the electron mass, \( m \), we shall take \( p_p = E_p, p_e = E_e \), and \( p'_e = E'_e \), wherever possible.

Since we assume \( E_p \gg M \), we may replace the definition of the variable \( \Sigma_h \) in (1) by

\[
\Sigma_h := \frac{P \cdot (P' - P)}{E_p}.
\]

This allows us to similarly reexpress the definitions (2) of the kinematic variables \( x_\Sigma \), \( y_\Sigma \), and \( Q^2_\Sigma \) through scalar products of four-momenta via

\[
y_\Sigma = \frac{P \cdot (P' - P)}{P \cdot (P' - P - p'_e)},
\]

\[
Q^2_\Sigma = \frac{4(p \cdot p')(P \cdot P')}{S},
\]

\[
x_\Sigma = \frac{Q^2_\Sigma}{y_\Sigma S},
\]

where

\[
S = 2P \cdot P.
\]

One important thing to note here is the nonlinear dependence of the kinematic variables \( x_\Sigma \) and \( Q^2_\Sigma \) on the energy and scattering angle of the outgoing lepton, while it is linear in the electron-only method (e-method).

As we are dealing with the kinematics of a process with real photon emission, it is convenient to define an invariant quantity \( \kappa \), that represents the energy of the outgoing photon in units of the energy of the incoming lepton, as measured in the rest frame of the incoming proton,

\[
\kappa := \frac{P \cdot k}{P \cdot p}.
\]

Energy and momentum conservation of the process (3) obviously requires \( 0 \leq \kappa < 1 \). In the special case of emission of the photon collinear to the incoming lepton, \( \kappa \) also represents the energy fraction of the initial lepton taken by the photon in the HERA lab frame.

Using momentum conservation (3) and relations (6) and (9), it is easy to see that

\[
P \cdot (P' - P) = (1 - \kappa) y_\Sigma P \cdot p = (1 - \kappa) y_\Sigma \frac{1}{2} S.
\]

\(^3\) As the tagged photon cross section represents a radiative correction to the DIS cross section \([7]\), one can in principle calculate the QED corrections to the former for any choice of kinematic variables with the help of a Monte Carlo event generator for DIS that properly implements the necessary higher order QED corrections to DIS. However, no generator exists for the calculation of QED corrections beyond leading logarithms, and the leading log generator KRONOS \([11]\) uses approximations for photon emission in the very forward direction that make it useless for the present task.