COMMUTATIVE HYPERCOMPLEX NUMBERS AND FUNCTIONS OF HYPERCOMPLEX VARIABLE: A MATRIX STUDY

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Abstract. Systems of hypercomplex numbers, which had been studied and developed at the end of the 19th century, are nowadays quite unknown to the scientific community. It is believed that study of their applications ended just before one of the fundamental discoveries of the 20th century, Einstein’s equivalence between space and time. Owing to this equivalence, not-defined quadratic forms have got concrete physical meaning and have been recently recognized to be in strong relationship with a system of bidimensional hypercomplex numbers. These numbers (called hyperbolic) can be considered as the most suitable mathematic language for describing the bidimensional space-time, in spite of some unfamiliar algebraic properties common to all the commutative hypercomplex systems with more than two dimensions: they are decomposable systems and there are non-zero numbers whose product is zero. With respect to the famous Hamilton quaternions, one can introduce the differential calculus for the hyperbolic numbers and for all the commutative hypercomplex systems; moreover, one can even introduce functions of hypercomplex variable.

The aim of this work is to study the systems of commutative hypercomplex numbers and the functions of hypercomplex variable by describing them in terms of a familiar mathematical tool, i.e. matrix algebra.

1. Introduction

It is well known that the application of the complex numbers to the solution of scientific problems goes beyond their algebraic introduction. As examples, we cite the use of complex variables for representing vectors in a plane and the use of functions of a complex variable to solve Laplace’s equation [1]. Hamilton introduced non-commutative quaternions for representing vectors in the three-dimensional space [2].

As far as hypercomplex numbers are concerned, we can introduce functions just for commutative systems [3]. In these systems, however, there are non-zero numbers whose product is zero [3], a property which is not found for real and complex numbers. Moreover, systems of hypercomplex numbers can be decomposed into subsystems of real and complex numbers [3]. For these reasons, the