On an Intrinsically Local Gauge Symmetric SU(3) Field Theory for Quantum Chromodynamics

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Abstract. The SU(3)$_{c}$ gauge theory for eight massless vector gauge fields is assimilated into the formalism which generated the intrinsic local gauge invariant U(1) and SU(2)$_{L}$ theories [30,31], thus establishing the theoretical structure for intrinsically local gauge invariant quantum chromodynamics. Use is made of both the standard and split algebra of the octonions, the last of the Hurwitz algebras, in devising the technology. Numerous interesting results are obtained, including an explanation of the inherently non-chiral nature of the strong and electromagnetic interactions in contradistinction to the weak interaction. The formalism’s novelty and universality compels contemplation of its potential ability to assimilate the gravitational interaction as well.

Keywords. SU(3)gauge theory, Local gauge invariance, Quantum field theory, Quantum chromodynamics, Strong force, Octonions, Hurwitz algebras, Normed division algebras, Particle physics, Yang–Mills theory, Standard model.

1. Introduction

This is the third$^{1}$ in a series of papers establishing a new formalism for deriving, and a new conceptual framework for conceiving, the fundamental interactions of nature. Just as the U(1) theory of quantum electrodynamics (QED) and SU(2)$_{L}$ theory were developed [30,31], herein is developed an intrinsically local SU(3) gauge invariant theory. Since SU(3)$_{c}$ theory in its unbroken form gives the theory of quantum chromodynamics (QCD) [4,11,12,19], the new formalism can be said to generate QCD in an intrinsically local gauge symmetric environment.

In addition to providing a unified and consistent mathematical rendition of the strong interaction, the simplicity and integrity of the formalism’s

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$^{1}$ See Ref. [30,31].

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approach makes it amenable for conceptually introducing the essential nature of the strong interaction to a broad level of studies within all potential fields of scientific and mathematical expertise. From a didactic standpoint this provides the formalism with a distinct advantage over other frameworks which concern the fundamental forces and which are often deemed inaccessible to all but a mathematical elite.

As an example, college-level mathematics and physics students are made well aware of the existence of the unique normed division (Hurwitz) algebras and their basic attributes. It would be fairly straightforward to introduce a section in the textbooks introducing the structures (operators and fields) of QCD through use of the current formalism, showing how the theory can be quickly built from the ground up within the confines of the Hurwitz algebras. Pure mathematics professors would assuredly find it satisfying—as it is in no doubt intriguing—to teach their students that the forms of the fundamental laws of physics are entirely dependent on the structure of the solely mathematically derived Hurwitz algebras. Further and importantly, such a didactic approach would also provide an excellent avenue for introducing the basics of the Clifford algebra—a mathematical arena becoming ever-important in fundamental physics—to all levels of students, as well as to scientists and mathematicians who do not typically practice in the area of theoretical particle physics.

So to begin, notice is first made that one of the only two algebras\(^2\) in the closed system of the four Hurwitz algebras\(^3\) with a non-trivial cross product\(^4\) plays a ubiquitous and necessary role in intrinsically gauging the \(U(1)\) and \(SU(2)\) theories \([30,31]\). The formalism for \(U(1)\) and \(SU(2)\) depends on and is mandated by the structure of the Hurwitz algebra \(Q\) along with its operator technology, including the operator coupling equation \([30]\)

\[
(v_0, v)(w_0, w) = (v_0 w_0 - v \cdot w, v_0 w + v w_0 + v \times w).
\]

This coupling equation exists for \(O\) as well \([1,7,9,25]\). Again, \(O\) is the only Hurwitz algebra other than \(Q\) with a non-trivial cross product. Further, the QCD \(SU(3)\) group is a subgroup of \(G_2\) \([1,18,20,22,25]\), which is the automorphism group of \(O\) \([1,6,10,20]\): \(SU(3)_c \subset G_2\). Lastly, QCD has been formulated in terms of \(O\) \([4,13,23]\).

These lines of thought lead to considering the octonion algebra as a structure for generalizing the formalism used for \(U(1)\) and \(SU(2)\) in order to generate an intrinsically local \(SU(3)\) gauge invariant Lagrangian for QCD.

### 2. The Octonions with Gell–Mann Basis

The goal of the current paper is not to frame QCD in terms of the octonions. This has already been achieved \([4,13,23]\), the results of which will be used herein.

\(^2\) The two algebras being the quaternions \(Q\) and octonions \(O\) \([1,18,20,25]\).

\(^3\) The four being \(\{\mathbb{R}, \mathbb{C}, \mathbb{Q}, \mathbb{O}\}\) \([20]\).

\(^4\) The \(\text{dim}(3)\) cross product for \(Q\) and the \(\text{dim}(7)\) cross product for \(O\) \([1,3,7,22]\).