Effect of Vertical Modulation on the Onset of Filtration Convection

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Abstract. The present paper examines the effect of vertical harmonic vibration on the onset of convection in an infinite horizontal layer of fluid saturating a porous medium. A constant temperature distribution is assigned on the rigid boundaries, so that there exists a vertical temperature gradient. The mathematical model is described by equations of filtration convection in the Darcy–Oberbeck–Boussinesq approximation. The linear stability analysis for the quasi-equilibrium solution is performed using Floquet theory. Employment of the method of continued fractions allows derivation of the dispersion equation for the Floquet exponent $\sigma$ in an explicit form. The neutral curves of the Rayleigh number $Ra$ versus horizontal wave number $k$ for the synchronous and subharmonic resonant modes are constructed for different values of frequency $\Omega$ and amplitude $A$ of vibration. Asymptotic formulas for these curves are derived for large values of $\Omega$ using the method of averaging, and, for small values of $\Omega$, using the WKB method. It is shown that, at some finite frequencies of vibration, there exist regions of parametric instability. Investigations carried out in the paper demonstrate that, depending on the governing parameters of the problem, vertical vibration can significantly affect the stability of the system by increasing or decreasing its susceptibility to convection.

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1. Introduction

Free convection in porous media is frequently encountered in nature and in engineering environments. The study of this phenomenon is of practical importance in many areas such as geophysics, oceanography, ecology, chemistry and metallurgy. Specific areas of application range from the flow of groundwater to oil recovery, energy storage, chemical reactor engineering and building insulation, to name a few. Furthermore, this phenomenon has practical importance for bone tissue engineering, which is an important area of current research in biomedicine.

The onset of convection in a horizontal layer of fluid saturating a porous medium was first investigated by Horton and Rogers [1], and independently by
Lapwood [2] using linear stability analysis. The results obtained by these authors were later confirmed experimentally by Elder [3], Bories [4], Katto and Masuoka [5] and many other authors. Theoretical investigation of thermosolutal convection in an infinite horizontal layer of a porous medium was first performed by Nield [6]. Beck [7] analytically studied convection in a finite rectangular box of fluid saturating a porous medium heated from below. Lyubimov [8] examined thermal filtration convection in a two-dimensional enclosure and showed that at supercritical values of the Rayleigh number, the problem has an infinite number of stationary solutions for any given shape of the region. The origin of this phenomenon was explained later in the co-symmetry theory developed by Yudovich [9, 10]. For a comprehensive review of the literature on free convection in porous media the reader is referred to the book by Nield and Bejan [11].

In addition to convection in porous media with no external force applied, attention has been given to convective instability in the presence of time-dependent body forces, one of which is vibration. The time-dependent gravitational field is of interest in space laboratory experiments, crystal growth, petroleum production and other areas. It also plays an important role in large-scale atmospheric convection.

The onset of convection in a region of fluid saturating a porous medium subjected to high-frequency vibration of arbitrary direction has been examined by Zenkovskaya [12, 13] using the averaging method. Malashetty and Padmavathi [14] asymptotically analyzed the linear stability of a horizontal fluid-saturated porous layer heated from below for the case of small-amplitude gravity modulation. Bardan and Mojtabi [15] studied numerically and analytically convection in a rectangular saturated porous cavity heated from below and subjected to high-frequency vibration.

The purpose of the current work is to investigate the effect of vertical modulation of arbitrary frequency on the onset of convection in a horizontal layer of fluid saturating a porous medium. It is of our goal to understand how varying the frequency and amplitude of vibration affects the stability of the quasi-equilibrium state of the system. Previous studies of this problem were restricted to the case of high frequency and small amplitude of vibration due to the limitations of the asymptotic methods used. The present work demonstrates the effectiveness of the method of continued fractions, which eliminates these restrictions and enables consideration of vibration of arbitrary frequency and amplitude.

The structure of the paper is as follows. Section 2 describes the mathematical model and the mechanical quasi-equilibrium solution to the system. Linear stability analysis of this quasi-equilibrium solution is performed in Section 3 using Floquet theory. Employment of the method of continued fractions allows derivation of the dispersion equation for the Floquet exponent \( \sigma \) in an explicit form. The case of high-frequency vibration of the layer is considered in Section 4 using the averaging method and the case of low-frequency vibration is considered in Section 5 using the WKB method. Section 6 presents the results of numerical calculations.