A Nonlinear Programming Technique for the Interpretation of Self-potential Anomalies

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Abstract — Using Frank and Wolfe's algorithm, a new interesting nonlinear programming technique has been developed in an attempt to estimate the geometric shape factor of a buried polarized body from a residual self-potential anomaly. Furthermore, the depth, the polarization angle and the electrical dipole moment have also been derived. This algorithm is noted to be robust and its application to SP data converges rapidly towards the optimal solution. The developed technique is tested through studying synthetic data with and without random noise. As a result, the near agreement between the model geometric shape factor and the evaluated one is well recognized. The validity of this proposed technique is tested on a field example from the Ergani Copper district, Turkey. The superiority of the nonlinear programming technique over other recently published methods is shown.

Key words: Self-potential anomalies, polarized structures, SP interpretation, mathematical programming.

Introduction

The self-potential (SP) method is based on the measurement of naturally occurring potential differences generated mainly by electrochemical, electrokinetic and thermoelectric sources. The SP method enjoys wide applications in engineering problems, groundwater investigations, the determination of subsurface temperature distributions and mineral investigations related to sulfides and graphite. The SP anomaly expression produced by some polarized geometric structures can be represented by a continuous geometric function which depends on the variables of geometric shape factor, depth, polarization angle and electrical dipole moment. Several quantitative methods have been developed by many researchers for interpreting SP anomalies: Yungul (1950), Roy and Chowdhury (1959), Paul (1965), Paul et al. (1965), Banerjee (1971), Bhattacharya and Roy (1981), Atchuta Rao and Ram Babu (1983), Meiser (1962), Rao et al. (1970), Sundarajan et al. (1998), Shalivahan et al. (1998), and Abdelrahman and Sharafeldin

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(1997). However, most of these interpretation methods necessitate a knowledge of the geometry (geometric shape factor) of the anomalous body, i.e., whether the source is a sphere or a cylinder. In the present paper, a nonlinear programming approach applying Frank and Wolfe’s algorithm is developed in terms of a geometric shape factor. The accuracy of the results obtained by the nonlinear programming approach depends upon the accuracy to which the residual anomaly can be represented from the observed self-potential data and the suitability of the model to the polarized object. The validity of this interpretation method is tested on theoretical examples and on a field example from the Ergani Copper district, Turkey.

Frank and Wolfe’s Algorithm for the Inversion of SP Data

Frank and Wolfe’s algorithm is commonly used for solving nonlinear problems (Steuer, 1986). In the present paper, this algorithm is adapted to be used for the inversion of SP measurements.

Let \( f : \mathbb{R}^n \rightarrow \mathbb{R} \) be a numerical function to be minimized over the subset \( A \subseteq \mathbb{R}^n \) which is convex and compact in the real space \( \mathbb{R}^n \). In a mathematical form the problem can be described as follows:

\[
\begin{align*}
\text{Min } f(x) \\
\text{Subject to } x \in A
\end{align*}
\]

To solve this problem, Frank and Wolfe’s algorithm functions as follows:

- Start with the initial point \( x^0 \in A \)
- Iterations \( k \)

(i) To determine the feasible (displacement) direction the following linear problem should be solved:

\[
\begin{align*}
\text{Max}[\nabla f(x^k)]^T x \\
x \in A
\end{align*}
\]

where \( \nabla f \) denotes the gradient of the function \( f \) and \( T \) denotes the transpose. Let \( y^k \) be an optimal solution. The feasible direction is defined as follows:

\[
d^k = y^k - x^k.
\]

(ii) Linear search in the feasible direction \( d^k \):

solving the following unidimensional problem:

\[
\begin{align*}
\text{Min } f(x^k - \lambda d^k) \\
0 \leq \lambda \leq 1
\end{align*}
\]

Let \( \lambda_k \) be an optimal solution. Calculating \( x^{k+1} = x^k - \lambda_k d^k \)

(iii) Stopping criteria. If verified end otherwise loop to step (i).