Interevent Time Distribution in Seismicity: A Theoretical Approach

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Abstract—This paper presents an analysis of the distribution of the time \( \tau \) between two consecutive events in a stationary point process. The study is motivated by the discovery of unified scaling laws for \( \tau \) for the case of seismic events. We demonstrate that these laws cannot exist simultaneously in a seismogenic area. Under very natural assumptions we show that if, after rescaling to ensure \( Et = 1 \), the interevent time has a universal distribution \( F \), then \( F \) must be exponential. In other words, Corral’s unified scaling law cannot exist in the whole range of time. In the framework of a general cluster model we discuss the parameterization of an empirical unified law and the physical meaning of the parameters involved.

Key words: Statistical seismology, point processes, self-similarity.

1. Introduction

Recently Bak et al. (2002) and Corral (2003a,b) suggested a new scaling law for seismic events on the phase space location-time-magnitude. The Corral’s version of this law looks as follows: The distribution density for time \( \tau \) between two consecutive events of magnitude \( m > m_c \) has the form

\[ p_\tau(t) = \lambda f(\lambda t), \]  \hfill (1)

where \( \lambda \) is the rate of events with \( m > m_c \) in a given area \( G \), while \( f \) is a universal function that is independent of the choice of \( G \) and cutoff magnitude \( m_c \). The relation (1) is astonishing, being tested (as it has been by Corral, 2003b) for a very wide range of \( m_c \) (between 2 and 7.5), for seismic regions \( G \) of very different linear size \( L \) (between 20 km and the size of the Earth), as well as for different catalogs, both regional and global ones, and different time periods.

The parameterization of \( f \) seems not yet to have settled down. According to Corral (2003b):

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\end{itemize}
\[ f(x) = cx^{\gamma-1} \exp(-x/a) \quad (2) \]

in the region \( x \geq 0.05 \) with \( \gamma = 0.74 \pm 0.05 \) and \( a = 1.23 \pm 0.15 \). The pioneering work (BAK et al., 2002) uses the parameterization (2) for the whole range of \( x \) with \( \gamma = 0.1 \) (see refined estimates in CORRAL, 2003a). This allows the behavior of \( f(x) \) about zero to be interpreted in terms of the Omori law.

The subsequent discussion strives to answer the following questions.

What is the behavior of the distribution of \( \tau \) near 0 and \( \infty \) in stochastic models of seismicity? These parts of the distribution of \( \tau \) usually dominate the log-log representation and therefore are important for understanding the unified law in the framework of the classical models. The next question is: what is the physical meaning of the parameters \( \gamma \) and \( a \) in (2)? Finally, assuming the form of \( f \) to be universal for \( \tau \), what should it be?

The answers to these questions are contained in Sections 2–4. All proofs of the main results are collected in the Appendix.

2. A Poisson Cluster Model: The Asymptotics of \( p_\tau(t) \)

Earthquakes frequently form anomalous clusters in spacetime. The largest event in a cluster is termed the main event. The events that occurred before and after the main event in a cluster are called fore- and aftershocks, respectively. It is assumed in a zero approximation that main events constitute a time-uniform Poisson process. That assumption is widely employed in seismic risk studies.

Aftershocks dominate clusters both as regards their number and duration. Their rate as a function of time is described by the Omori law:

\[ n(t) = ct^{-p}, \quad t > t_0, \quad (3) \]

where \( t_0 \) is small. Relation (3) holds fairly well during the first few tens of days (up to a year) with the parameter \( p = 0.7 – 1.4 \) (UTSU et al., 1994) At large times the value of \( p \) becomes greater, occasionally significantly so, making \( n(t) \) decay in an exponential manner. Taken on the whole, background seismicity and spatial interaction do not allow reliable conclusions to be drawn for the Omori law at large times. Cases in which (3) holds during decades are unique (UTSU et al., 1994).

Following the above description, we consider the following model for seismic events in time. The spatial and magnitude components of events are disregarded for simplicity of reasoning. Let \( \{x_i\} \) be a homogeneous Poisson point process on a line with rate \( \lambda^* \). It is an analogue of main events. Let \( N_0(dt) \) be an inhomogeneous point process with rate \( \delta(t) + \lambda_0(t) \). Here, \( \delta \) is the delta function, while the presence of \( \delta(t) \) means that the event \( t = 0 \) belongs to \( N_0 \). The notation \( N_0(\Delta) \) defines the number of events \( N_0 \) in the interval \( \Delta \). We will assume that