Bernhard Riemann
Remarks on his Life and Work

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Abstract. The first part contains a few facts about Riemann taken from correspondence (of Riemann and of Dedekind), and then describes Riemann’s letter to Weierstrass concerning his paper on the distribution of primes. It is followed by a brief outline of the paper itself.

The second part deals with his course of Wintersemester 1858/59 on the hypergeometric series, which anticipated by a decade important work of L. Fuchs and H.A. Schwarz. The course also contains Riemann’s ideas on what we would now call finite dimensional local systems on $P^1 \setminus S$ where $S$ is finite.

Mathematics Subject Classification (2010). 01A70, 01A55, 11N05, 11S40, 34M03, 34M35.

Keywords. Riemann’s life, zeta function, distribution of primes, monodromy.

Riemann’s collected works [1] contain a biography (of just 17 printed pages) written by his friend and colleague Richard Dedekind. Dedekind knew Riemann well, and as he was also careful and meticulous, details of Riemann’s life are set forth clearly and accurately.

Georg Friedrich Bernhard Riemann was born on the 17th September, 1826 in Breselenz in the state of Hanover, the second of six children. He had an older sister Ida, three younger sisters Clara, Helene, Marie, and a younger brother Wilhelm. His parents were Friedrich Bernhard Riemann and Charlotte Ebell. He started school in Hanover in 1840, and moved, in 1842, to Lüneburg for four years of study at the Gymnasium there before entering the University of Göttingen in 1846. He spent the two years 1847–1849 in Berlin before returning to Göttingen in 1849.

He submitted his Doctoral Dissertation in 1851; in 1854, he completed both the written thesis and the Inaugural Lecture for his Habilitation (necessary to become a Privatdozent, an unpaid lecturer who received fees from the students attending his lectures).

Gauss died in 1855 and Dirichlet came to Göttingen as his successor. Riemann was not appointed an extraordinarius (Außerordentlicher Professor, roughly, an Assistant Professor) as some had expected. Instead he was granted a small stipend.

This text is based on a talk at the RISM conference in Verbania, Intra, Italy, April 19–24, 2009.
1857, he was appointed extraordinarius, but at a smaller salary than was customary for the position. Finally, in 1859, following the death of Dirichlet, he became a (Full) Professor.

In June 1862, Riemann married Elise Koch, a friend of his sisters. They had a daughter in December, 1863, whom they named Ida after his older sister.

Between November, 1862 and October, 1865, Riemann made two extended trips to Italy for health reasons. He returned to Italy for a third time in June, 1866, to a village near Intra on the Lago Maggiore. But his health worsened, and he died on the 20th July, 1866 on the shores of Lago Maggiore, not quite 40 years old.

These then are the bare facts concerning Riemann’s life. Dedekind’s biographical sketch fleshes these dates out to a certain extent with some personal details taken largely from Riemann’s letters to his family, as well as information about his mathematical work. Of course, one learns more about Riemann’s personality from correspondence, both his own letters and those of others who knew him. Many of his mathematical ideas remained unpublished during his lifetime. Dedekind published those manuscripts of Riemann’s that he judged to be in final form shortly after Riemann’s death. Heinrich Weber, and Max Noether and Wilhelm Wirtinger edited and included several more works of Riemann in the two editions of his collected works. However, other ideas of importance remained in the notes of the courses Riemann gave. I shall have something to say about one of these courses later on.

Dedekind’s biography also contains information about Riemann’s school years (taken from his many letters home). At the request of E. Schering after Riemann’s death, two of Riemann’s teachers G.H. Seffer and C. Schmalfuss wrote Schering at the end of November, 1866. It becomes clear from these sources that Riemann always had difficulty in social contact with people outside his immediate family; moreover, he had the greatest trouble in putting his thoughts into words, and essays cost him a great deal of effort. Needless to say this affected his published output very adversely. Dedekind, in a letter to his sister Mathilde in 1857, gives some idea of the consequences he suffered when he had to write up his ideas under some time pressure. This letter was written shortly after Riemann completed his great paper on Abelian functions. Dedekind sent Riemann to his family house in Hartzburg in the company of an acquaintance, Ritter, so that Riemann could relax. Dedekind says (my translation) “…but his lonely life and the added physical distress have made him most hypochondriacal and mistrusting of people, even when he seems outwardly friendly.”

Even in his early years in Göttingen this mistrust seems to have manifested itself. In two letters written to his father from Berlin in 1847, he speaks about his relations with Eisenstein. Riemann had attended Eisenstein’s lectures on elliptic functions and, according to Dedekind, had spent quite a bit of time discussing the foundations of the theory of functions of a complex variable with Eisenstein. But these two letters to his father indicate that their relationship had soured, at least on Riemann’s side. In the first letter (23rd July, 1847) he says that he had had a

1“hypochondrisch”, in the German original, seems to have also some connotation of depression.