BIT ERROR PROBABILITY AND BIT OUTAGE RATE IN CHAOS COMMUNICATION*

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Abstract. This paper investigates the notion of the probability of bit error (PBE) and its distribution in chaos-based communication systems; these are seen as being the fundamental quantities to both the well-known bit error rate (BER) and the new concept in chaos communications of bit outage rate (BOR). The form of the distribution illustrates the degree to which bit error rate is a stable representation of performance. Bit outage rate is another measure of performance which gives practically helpful information about bit error. For a simple coherent chaos-shift-keying system the distribution of bit error probability is derived exactly, and theoretically exact formulas for the bit outage rate and bit error rate are presented. Two specific cases are developed to obtain useful qualitative and quantitative information. The cases concern independent Gaussian spreading, as a lower benchmark and logistic map spreading, as typical of effective chaotic spreading. Comparisons are obtained between these spreading distributions and between different extents of their spreading, calibrated against per bit signal to noise ratio. A general conclusion is that bit outage and bit error rates are complementary measures of performance.

Key words: Bit error rate, bit outage rate, chaos-shift keying communication systems, distribution of bit error probability.

1. Introduction

The performance assessment of chaos-based communication systems has in the past mainly been limited to bit error rate (BER); by being a rate, it is an average over a distribution of many bits, and the relevant probability distribution has not previously been investigated. Another general idea of performance in

* Received February 15, 2005; revised May 30, 2005; This work was supported in part by the UK Engineering and Physical Sciences Research Council (EPSRC) under Grant GR/M74795 and by the Shiga University Sabbatical Fund.

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communication systems is bit outage rate (BOR), the main focus of this paper, and not previously discussed for chaos-based communication systems, it would seem. The concept tries to capture the rate of transmission for which a bit error is likely, according to a probability quantification of “likely,” rather than the average rate at which errors actually occur. Thus it refers to the probability of bit error (PBE) directly and the rate at which this has a specified probability. The aim of this paper is to set out an approach to the exact calculation of the probability of bit error and its accurate approximation. This is then linked to both bit error rate and bit outage rate, developing the authors’ earlier exact approach to bit error rate [12]. Exemplification will be with coherent chaos-shift-keying systems employing their correlation decoders and taking both independent Gaussian and logistic map spreading. A key theoretical concern has to be with the distribution of bit energy, typically not constant with chaos-based systems, as first noted in [6], and which features in PBE. With admittedly unrealistic independent Gaussian spreading, the exact distribution of bit energy is chi-squared, but it is not exactly tractable in other cases. However, for the more practically relevant chaotic map spreading, the bit energy distribution can be approximated by a scaled chi-squared distribution [15], and this allows much easier analytical calculation of bit outage rates.

The aims of this paper are motivated by the earlier work on chaos communication systems, those employing chaotic waves in place of conventional sinusoidal waves. This work mostly gave noise performance assessment based on simulation and approximate analytics, rather than exact theory. Among the earlier work, a trio of papers, [7], [8], and [9], introduced and developed the basic ideas of chaos communication, while in [1], [2], and [14] there were the first developments of their noise performance approximate analytics. The monograph [11] coordinated much existing research in the area and also introduced further topics. There is another area of research, seen from [3], concerned with optimal maximum likelihood decoders.

The general mathematical approach here is to use discrete-time base-band models which are designed to be equivalent to continuous-time random process models as far as performance studies are concerned. They simplify much of the traditional continuous-time mathematical basis of the communications modeling.

2. Probability of bit error and CSK systems

Although the basic ideas underlying this paper are not specific to any particular chaos-based spread spectrum communication system, the well-known chaos-shift-keying (CSK) system in its binary (antipodal) bit and coherent form will be used as the basis of the development.

Suppose the CSK system uses chaotic spreading sequences \( \{X_i\} \) which are generated by the chaotic map \( \tau(z), \) \( -c < z < c, \) with a symmetric invariant distribution, thus of mean 0, and with variance \( \sigma^2_{X} \); there is no lack of generality