LINEAR UNBIASED STATE ESTIMATION WITH RANDOM ONE-STEP SENSOR DELAY *

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Abstract. Sensor delay and observation uncertainty often occur in modern computer-based systems, e.g., when the measurement is transmitted to a remote controller through a network medium. In this paper, we revisit the Kalman filter design problem for a stochastic dynamic system with random one-step sensor delay, and derive the optimal unbiased state estimation algorithm. Both full- and reduced-order filters are studied, and the results compare favorably with those of the existing algorithms in examples via simulation.

Key words: Filter design, state estimation, random time delay, uncertain observation.

1. Introduction

In networked control systems, sensory information is transmitted through network media, giving rise to random sensor delay and observation uncertainty, which are primary sources of instability and performance degradation in networked feedback systems [7], [2], [9], [6], [13], [4], [3]. Sensor delay is due to the presence of a communication channel, which typically has limited capacity and bandwidth, and is shared by many other users/tasks [9]. Because of these communication-induced constraints, filtering and control systems designed by conventional methods that ignore communication constraints may not work as desired, and sometimes may even fail to satisfy the performance and stability requirements. Thus, in networked system analysis and design, it is important to

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incorporate random sensor delay and such—this motivates the present study on state estimation involving delay and observation uncertainty.

In the literature, Yaz and Ray performed the first study on linear unbiased state estimation for dynamic systems with one-step sensor delay and presented full- and reduced-order estimators by reformulating the state estimation problem into parameter design in the filtering problem [11], [12]. Wang et al. studied (suboptimal) filter design with random sensor delay subject to variance constraints [8]. However, the work in [11], [12] was deficient: In the algorithm derivation, the reformulated measurement equation contained a colored noise, which was overlooked and treated as a white noise; this renders the filters reported in [11], [12] nonoptimal.

The contributions in this paper are as follows. We treat the colored noise by augmentation and transfer the filter design problem into a form tractable by existing filter design methods (Section 2). Based on the transformed model, we derive both full- (optimal) and reduced-order linear unbiased estimators with different dimensions (Section 3). Finally, we present extensive simulation results to demonstrate that our proposed algorithms outperform the ones reported in [11], [12] (Section 4).

2. Problem formulation

In this section, we will set up the system model to be studied, discuss the assumptions, and introduce a model transformation.

2.1. Model description

Consider a class of discrete-time linear dynamic systems modeled by the state equation

\[ \tilde{x}(k + 1) = \Phi(k)\tilde{x}(k) + F_1(k)w(k), \]  

where \( \tilde{x}(k) \in \mathbb{R}^n \) is the state vector to be estimated, \( \Phi(k) \in \mathbb{R}^{n \times n} \) and \( F_1(k) \in \mathbb{R}^{n \times m} \) are known real matrices, and \( w(k) \in \mathbb{R}^m \) is a (white) process noise sequence with zero mean and known covariance

\[ E\left\{ w(k)w^T(j) \right\} = \delta_{kj} Q(k). \]  

Here, \((\cdot)^T\) denotes the transpose; the process noise covariance \( Q(k) \) satisfies \( \text{Trace} \ Q(k) < \infty \), and \( \delta_{ij} \) is the Kronecker delta function. The measurement is

\[ \tilde{y}(k) = \tilde{C}(k)\tilde{x}(k) + F_2(k)v(k), \]  

where \( \tilde{C}(k) \) is an unknown matrix.