WEAKLY HYPERBOLIC ACTIONS OF KAZHDAN GROUPS ON TORI

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Abstract. We study the ergodic and rigidity properties of weakly hyperbolic actions. First, we establish ergodicity for $C^2$ volume preserving weakly hyperbolic group actions on closed manifolds. For the integral action generated by a single Anosov diffeomorphism this theorem is classical and originally due to Anosov.

Motivated by the Franks/Manning classification of Anosov diffeomorphisms on tori, we restrict our attention to weakly hyperbolic actions on the torus. When the acting group is a lattice subgroup of a semisimple Lie group with no compact factors and all (almost) simple factors of real rank at least two, we show that weak hyperbolicity in the original action implies weak hyperbolicity for the induced action on the fundamental group. As a corollary, we obtain that any such action on the torus is continuously semiconjugate to the affine action coming from the fundamental group via a map unique in the homotopy class of the identity. Under the additional assumption that some partially hyperbolic group element has quasi-isometrically embedded lifts of unstable leaves to the universal cover, we obtain a conjugacy, resulting in a continuous classification for these actions.

1 Introduction

In this article we investigate the notion of weak hyperbolicity for group actions first introduced in [MaQ]. For linear representations, weak hyperbolicity requires that there are no nontrivial subrepresentations for which all eigenvalues of all group elements have modulus one. Weak hyperbolicity for a group action on a closed manifold is a differential-geometric version

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of that for representations. The classical weakly hyperbolic dynamical systems, i.e. weakly hyperbolic actions of the integers, correspond to the well-understood class of Anosov diffeomorphisms. With this in mind, we shall draw conclusions about weakly hyperbolic actions analogous to well-known results describing Anosov diffeomorphisms. For example, Theorem 3.5 establishes the ergodicity of weakly hyperbolic actions, generalizing Anosov’s work on ergodicity of Anosov diffeomorphisms [A, Thm. 4]. The proof of Theorem 3.5 uses ideas from the well-known Hopf argument (see [BuPSW, §2.1] for a description). Indeed, the proof is geometrically based on the presence (and accessibility) of stable foliations, and technically depends on the absolute continuity of these foliations. Perhaps new here is the fact that the core of this argument for ergodicity lies in the use of a regularity result from [RT] relating Sobolev classes of functions measured tangentially with respect to absolutely continuous foliations to global Sobolev classes of functions.

The remainder of this work is motivated by that of Franks and Manning, [Fr], [M], on the classification of Anosov diffeomorphisms on tori. One may ask more generally whether weakly hyperbolic actions on tori are classifiable. When the acting group is a higher rank lattice, this question falls into Zimmer’s program for classifying volume preserving ergodic actions of higher rank lattices on closed manifolds [Z]. Our Theorem 5.2 is an analogue of Manning’s contribution to the classification of Anosov diffeomorphisms on tori. Roughly speaking, it asserts that weak hyperbolicity is inherited by the action on the fundamental group when the acting group is a higher rank lattice. The proof uses a rigidity property of the higher rank lattice, specifically that such groups have Kazhdan’s property (T), to first draw conclusions about the action in the measurable category and then uses the dynamical assumption of weak hyperbolicity to bootstrap regularity from measurable to continuous. Theorem 5.2 confirms what is suggested to be true by Margulis and Qian in [MaQ]. Therein, and more generally in [FW], the analogue of Frank’s contribution is proven in the higher rank lattice setting. Their result coupled together with the complementary Theorem 5.2 establishes that all $C^2$ volume preserving weakly hyperbolic actions of higher rank lattices on tori, covered by an action on $\mathbb{R}^n$, are continuously semiconjugate to the affine action coming from the fundamental group (after possibly passing to a finite index subgroup of the lattice). If, in addition, some group element acts by a partially hyperbolic diffeomorphism with a quasi-isometric (in the universal cover) unstable foliation, then we show this semiconjugacy is injective, providing