An augmented Lagrangian optimization method for inflatable structures analysis problems

Abstract This paper describes the development of an augmented Lagrangian optimization method for the numerical simulation of the inflation process in the design of inflatable space structures. Although the Newton–Raphson scheme was proven to be efficient for solving many nonlinear problems, it can lead to lack of convergence when it is applied to the simulation of the inflation process. As a result, it is recommended to use an optimization algorithm to find the minimum energy configuration that satisfies the equilibrium equations characterizing the final shape of the inflated structure subject to an internal pressure. On top of that, given that some degrees of freedom may be linked, the optimum may be constrained, and specific optimization methods for constrained problems must be considered. The paper presents the formulation and the augmented Lagrangian method (ALM) developed in SAMCEF Mecano for inflatable structures analysis problems. The related quasi-unconstrained optimization problem is solved with a nonlinear conjugate gradient method. The Wolfe conditions are used in conjunction with a cubic interpolation for the line search. Equality constraints are considered and can be easily treated by the ALM formulation. Numerical applications present simulations of unconstrained and constrained inflation processes (i.e., where the motion of some nodes is ruled by a rigid body element restriction and/or problems including contact conditions).

Keywords Augmented Lagrangian method · Conjugate gradients method · Inflatable structures · Contact

1 Introduction

Lightweight inflatable space structures are becoming very attractive because they can meet structural requirements for space applications at a much lower cost than conventional space structures. The applications of inflatable technology are numerous: It can be used for solar sails, thermal shields, pressurized habitats in space, solar arrays, antenna reflectors, and telescope mirrors. The significant advantage of inflatable technology with respect to conventional mechanical technology is the small volume necessary for the storage of the space structure during its launch and the large size after its deployment in orbit. However, inflatable space structures also have a serious drawback. The development and the test of prototypes in space is prohibitively expensive and ground tests, even if performed in a vacuum chamber, poorly represent the real behavior of this type of structure in space. For this reason, the numerical analyses of inflatable space structures are becoming absolutely necessary.

Although the Newton–Raphson scheme was proven to be efficient for solving many nonlinear problems, it can lead to large oscillations and lacks of convergence when it aims to find the nodal positions of the discretized inflatable structure for an equilibrium state.

To circumvent this difficulty, an explicit scheme can be selected (Troufflard et al. 2005; Oñate et al. 2005) that provides some inertia avoiding large movements of the nodal positions. Because it requires tuning of damping parameters, undesirable dynamical effects can occur, leading to a nonrealistic inflation process simulation. To solve this problem, an optimization approach was sometimes used in the literature (Tielking and Feng 1974; Little 1987; Baginski and Ramamurti 1995; Baginski and Schur 2003; Bouzidi and Le Van 2004) to find the minimum energy configuration that satisfies the equilibrium equations characterizing the final shape of the inflated structure subject to an internal pressure. In addition, given that some degrees of freedom may be linked and that resource constraints such as a given gas volume may be considered in the design problem, the optimum may be constrained, and constrained optimization techniques must then be used.
The paper presents the formulation and the augmented Lagrangian method (ALM) developed in the industrial implicit nonlinear finite element software SAMCEF Mecano (http://www.samcef.com) for inflatable structures analysis problems. The related quasi-unconstrained optimization problem is solved with a nonlinear conjugate gradients method. The Wolfe conditions are used in conjunction with a cubic interpolation for the line search. Equality constraints are considered and can be easily treated by the ALM formulation. Numerical applications present simulations of unconstrained and constrained inflation processes (i.e., where the motion of some nodes is ruled by a rigid body element (RBE) restriction and/or problems including contact conditions).

It should be noted that the current development is a part of a general project dedicated to the numerical simulation of inflatable structures (Bruyneel et al. 2005) that also includes developments on fluid modeling and material behavior.

2 The design problem under consideration

The design problem consists in finding the final shape of an inflatable structure in equilibrium with an internal pressure $P$. Because the problem is discretized with finite elements, the nodal positions $q$ are the unknowns. Constraints can appear in the problem, that is, some degrees of freedom can be linked together to simulate RBE, or contact conditions can be imposed. An alternative problem not considered here consists in finding the final shape for a given gas volume $V$ (Fig. 1).

3 Comparison of the Newton–Raphson and the optimization approaches

Instead of directly solving a system of nonlinear equations for finding the nodal positions $q$ at the equilibrium state for a given internal pressure, an optimization algorithm can be used for finding this solution.

In the case of unconstrained optimization problems, one looks for the minimum of the total potential energy $\pi(q)$ over the nodal positions (1). The related optimality conditions state that the objective function’s gradient must vanish at the solution. This condition is identical to imposing null residue for the equilibrium (2) where $F_{\text{int}}$ and $F_{\text{ext}}$ are the internal and external forces, respectively.

Optimization approach: $\min_q \pi \Rightarrow \nabla \pi = F_{\text{int}}(q) - F_{\text{ext}} = 0$ (1)

Newton - Raphson approach: $\frac{\partial \pi}{\partial q_i} = 0, i = 1, n \Rightarrow F_{\text{int}}(q) - F_{\text{ext}} = 0$ (2)

In the case of constrained optimization problems, the Karsuh–Khun–Tucker optimality conditions must be satisfied. The problem includes constraints that can be used for defining RBE in the model or contact conditions. If the $m$ (equality) constraints are noted $c_j(q)$, the corresponding augmented Lagrangian of the problem can be written as:

$$ L_a = \pi + k \sum_j \lambda_j c_j + \frac{p}{2} \sum_j c_j^2 $$ (3)

where $\lambda_j$ are the Lagrangian multipliers associated to the constraints. The parameters $k$ and $p$ are the scaling and regularization factors, respectively. Solving this problem as a system of equations, it comes that:

$$ \frac{\partial L_a}{\partial q_i} = 0, i = 1, n \Rightarrow \frac{\partial \pi}{\partial q_i} + k \sum_j \lambda_j \frac{\partial c_j}{\partial q_i} + p \sum_j c_j \frac{\partial c_j}{\partial q_i} = 0 $$ (4)

$$ \frac{\partial L_a}{\partial \lambda_j} = 0, j = 1, m \Rightarrow kc_j = 0 $$ (5)

and the Newton–Raphson solution procedure used in (2) is applicable here for solving the system of equations (4) and (5). On the other hand, the related optimization problem is the following:

$$ \min_q \pi \quad \text{subject to} \quad c_j = 0 \quad j = 1, m $$ (6)

Irrespective of the chosen optimization method, the process runs until the Karush–Khun–Tucker conditions are satisfied.