Concurrent material and structural optimization of hollow plate with truss-like material

Jun Yan · Gengdong Cheng · Ling Liu · Shutian Liu

Abstract In this paper, optimum stress distribution for hollow plates composed of linear cellular materials (LCMs), a kind of truss-like material, is investigated. To reduce the computational cost, we model the material as micropolar continua representation. Two classes of design variables, relative density, and cell-size distribution of truss-like materials are to be determined by optimization under given total material volume constraint. And the concurrent designs of material and structure are obtained for three different optimization formulations. For the first formulation, we aim at the minimization of the maximum stress that appears at the initial uniform design; for the second formulation, we minimize the highest stress within the specified point set. As the yield strength of truss-like material is dependent on the relative material density, we minimize the ratio of stress over the corresponding yield strength along the hole boundary in our third formulation, which maximizes the strength reserve and seems more rational. The numerical results for the three objectives validate the concurrent optimization method proposed in this paper. And the influence of ply angle (angle between the principle direction of material and the axes of the system’s coordinates) on the optimum result is discussed. The dependence of optimum design on finite element meshes is also investigated. An approximate discrete model is established to verify the method proposed in this paper, and the stress concentration near a hole is reduced significantly.

Keywords Truss-like material · Micropolar theory · Concurrent design of materials and structures · Stress optimization

1 Introduction

The ultra-light material with high stiffness and high strength has attracted more and more attention recently. The use of the new material for vehicles reduces structural weight and energy consumption, which greatly improve the quality of products in automobile, aerospace, and aircraft industries. Among various ultra-light materials, truss-like material has received increasing attention for its high stiffness–weight and strength–weight ratios in the past decade, and it has been used in more applications as a result of the rapid developments in manufacturing techniques (Brittain et al. 2001). Deshpande et al. (2001a and b) investigated the effective mechanical properties of the octet-truss lattice structured material, both experimentally and theoretically. Analytical and FE calculations of the elastic buckling and plastic yield surfaces were reported, and the criteria for construction of stretching-dominated cellular materials were proposed. Wallach and Gibson (2001) reported a combined experimental and finite element analysis (FEA) investigation on strength and stiffness of a truss plate and found that properties of truss materials were favorable compared with those of metallic foams. Wicks and Hutchinson (2001) showed that optimal truss panels were exceptionally more efficient for carrying bending and compression loads as compared to alternatives such as honeycomb-core sandwich panels or stringer stiffened plates. Hyun et al. (2003) used the finite element method to simulate the properties of panels with Kagomé and tetragonal cores under compressive and shear loading and
found that Kagomé core is more resistant to plastic buckling than tetragonal core under both compression and shear. Specially, truss-like materials have a combination of properties that make them very suitable for a wide range of applications such as ultra-light-weight structures, heat exchangers, energy absorption systems, vibration control, and acoustical scattering (Simone and Gibson 1997; Gibson and Ashby 1997).

Linear cellular materials (LCMs) are metal truss-like materials manufactured using a powder slurry extrusion process developed by Cochran et al. (2000, 2001). This manufacturing process allows complex cell arrangements and shapes to be fabricated at fine scale and, hence, provides a tremendous opportunity to tailor these materials for a given multifunctional application. Figure 1 gives typical microstructures from LCMs that are considered in the present research. In practical applications, holes are frequently made on structures composed of LCMs that bring about stress concentration around the hole and lead to damage and early failure. Therefore, it is of great interests to study how to optimize the design of structure and material distribution concurrently so as to reduce the stress concentration around the hole (Lipton 2002; Allaire et al. 2004).

In this paper, optimum stress distribution around a hole is investigated for a hollow plate composed of this kind of LCMs with micropolar continua representation to reduce the computational cost. Two classes of design variables, relative density and cell size distribution of LCMs, are to be determined by optimization under given total material volume constraint. And the concurrent designs of materials and structures are obtained for three different optimization formulations. For the first formulation, we aim at the minimization of the stress around the hole; for the second formulation, we minimize the highest stress within the specified points set. Because the effective yield strength of truss-like material is dependent on the relative material density, we minimize the ratio of stress over the corresponding yield strength in our third formulation, which maximizes the strength reserve and seems more rational. Numerical results for the three objectives validate the method proposed in this paper. And the influence of ply angle (angle between the principle direction of material and the axes of the system’s coordinate) on the optimum result is discussed. The dependence of optimum design on finite element meshes is also investigated. An approximate discrete model is established to verify the method proposed in this paper, and the axial stress in the beam element corresponding to point with stress concentration is found reduced significantly compared with the counterpart of homogeneous LCMs distribution.

2 Micropolar linear elastic theory for 2D LCMs

2.1 Representation of two-dimensional periodic LCMs as micropolar continua

In the straight approach, the overall properties of LCMs are evaluated via discrete modeling in which the individual cell walls are modeled as beams or rods. Such discrete model gives very accurate stresses and strains in cell walls; however, they are computationally expensive, thus, unacceptable for optimization of practical structures. Furthermore, in many applications, such accuracy is unnecessary. For example, in the phase of the product conception design and shape selection, it is accurate enough to represent the 2D LCMs as an equivalent continuum as opposed to an expensive discrete modeling. The equivalence reduces the amount of computation significantly while still guaranteeing the precision of analysis, thus, providing a solid basis for investigation of the optimum distribution of LCMs. There are large amount of literatures on this type of equivalence, among which the representative volume element (RVE) method and homogenization method (Pecullan et al. 1999; Guedes and Kikuchi 1990) are most representative. In all these literatures, truss-like materials are modeled as classical continuum that assumes only two translational degrees of freedom for each material point, the interaction between neighboring material points being via stress traction and the theory being local.

However, as pointed out by Fleck and Hutchinson 1997 and Adachi et al. 1997, for those cases in which the gradient of the strain is large, the classic continuum theory is not accurate enough. To capture this limited non-locality, independent rotational degrees of freedom and couple stresses in addition to the usual Cauchy-type stress at each material point have been introduced to enrich the classical theory. This theory is referred to as Cosserat or micropolar theory (Eringen 1999; Chen et al. 1998). Because the solid part of LCMs is distributed discretely in space and there are large gradient of strain around the hole, we need to model the LCMs as micropolar continuum to guarantee the sufficient computation precision, thus, providing a solid analysis basis for the optimization design. It is worth pointing out that the approach presented in this paper only