Structural optimization of elastic columns under stress corrosion conditions

M. Fridman* and M. Życzkowski

Abstract Optimal design of nonprismatic columns under concentrated axial force subject to corrosive environment is considered. The initial volume of a column is the design objective whereas the constraint refers to elastic buckling at a certain prescribed life-time with corrosive wear of this column taken into account. Plane-tapered and uniformly spatially tapered columns are optimized under the conditions of plane or spatial corrosion. Variational treatment leads to fourth-order Euler–Lagrange equations. The Chentsov method makes it possible to integrate these equations twice in an analytical manner and to reduce them to second-order equations; then numerical integration is employed. Final optimal shapes of the columns are shown both before corrosion and after corrosive wear corresponding to the prescribed life-time.

Key words column optimization, corrosion constraints, life-time

1 Introduction

Metal structures, in particular steel structures are often subject to corrosion due to an aggressive environment. In such cases, the optimal design of structural elements should allow for the effects of corrosion. These effects increase in time and hence design for a prescribed life-time \( t_\text{s} \) takes place.

Corrosion is the destruction of metal by chemical change, electrochemical change or physical dissolution (Evans 1960). A great variety of forms of corrosion may be encountered in engineering applications; they depend on the structural material, aggressive environment, temperature, state of stress in the structural element, etc. The most important forms are: general corrosion, localized and pitting corrosion, all appearing on the metal surface; intergranular, transgranular and layer corrosion, if the attack extends inwards. Moreover, conjoint action of destructive agents may be observed; in structural elements conjoint action with the stress state is particularly important, and particular forms of this action are called stress corrosion, corrosion cracking, and under variable loads – corrosion fatigue. In the present paper we consider only loadings constant in time and hence corrosion fatigue is not allowed for. Moreover we neglect the difficult problems of intergranular, transgranular and layer corrosion, the evolution of which is described mainly by diffusion equations; approximate estimation of the relevant effects will just be mentioned in Sect. 5. Thus we consider here general corrosion, localized and pitting corrosion appearing on the metal surface. In a reasonable approximation we assume that global effects of these corrosion types may be characterized by a surface layer of corrosive wear with thickness \( \delta \) and described by suitable evolution equations.

Many papers are devoted to analysis and optimization of structural elements under corrosion conditions. Numerous results are presented in the books by Ovchinnikov et al. (1995), Pochtman and Fridman (1997). The authors described \( \delta \) by evolution equations of the type

\[
\frac{d\delta}{dt} = f(t, \sigma_{\text{red}}, \delta, T),
\]

where \( t \) denotes time, \( T \) – absolute temperature, and \( \sigma_{\text{red}} \) – reduced stress according to a certain appropriate failure hypothesis. In particular, the authors compared three evolution equations: that proposed by Dolinsky (1967)

\[
\frac{d\delta}{dt} = \varphi(t)(1 + k\sigma_i),
\]

that by Gutman and Zaynullin (1984)

\[
\frac{d\delta}{dt} = v(t) \exp(\gamma\sigma_m),
\]
and that by Ovchinnikov (1979)

\[
\frac{d\delta}{dt} = v_0(1 + \xi \Phi),
\]

where \(\sigma_t\) denotes the Huber-Mises-Hencky stress intensity, \(\sigma_m\) – mean stress, \(\Phi\) – the specific elastic energy replacing here the reduced stress, \(\varphi(t)\) and \(v(t)\) are certain functions of time, \(k, \gamma, v_0\) and \(\xi\) – constants depending on the temperature \(T\).

In the present paper we consider the optimal design of nonprismatic columns under axial compression and the constraint of buckling in a corrosive environment. This problem was considered by Pochtman and Fridman (1997) and consulted by Zyczkowski (1955); the authors gave two particular solutions for Dolinsky’s law (2) with \(\varphi(t) = \text{const.}\). Here we give a more general approach and consider various combinations of corrosive wear and tapering of the columns. The paper is based on the following assumptions.

1. The column under the compressive force \(P\) is located in a corrosive medium. Global effects of corrosion are characterized by a surface layer of corrosive wear with the thickness \(\delta\). The evolution equation for \(\delta\) is assumed in Dolinsky’s form (2) with \(\varphi(t) = \text{const.}\). Here we give a more general approach and consider various combinations of corrosive wear and tapering of the columns. The paper is based on the following assumptions.

2. The minimal initial volume of the column, before the effects of corrosion, is the design objective.

3. The basic constraint refers to the buckling load at the moment \(t = t_*\) corresponding to the prescribed life-time of the column.

4. Buckling at \(t = t_*\) is assumed to be elastic, but with bending stiffness reduced in view of corrosive wear. Elastic-plastic buckling will not be considered.

5. Plane or spatial tapering of a nonprismatic column serves as the design variable.

The problem of the optimal design of columns for a certain prescribed life-time resembles that of optimal design for creep buckling (Życzkowski and Wojdanowska-Zając 1970-1972; Blachut and Życzkowski 1984; Wróblewski and Życzkowski 1989). However, some essential differences between these problems will be pointed out.

2 Stress corrosion of columns

2.1 General anisotropic corrosion of columns

In the case of elastic columns buckling is determined by the minimal cross-sectional moment of inertia, whereas the shape of the section is irrelevant. Conversely, most theories of elastic-plastic buckling introduce a certain dependence on the cross-sectional shape. In the case of columns in a corrosive environment the cross-sectional shape is even more important, since the length of the perimeter of the section affects directly the corrosive wear. In order to simplify the considerations we confine ourselves mainly to rectangular cross-section with the dimensions \(b\) and \(h\); some attention will also be paid to circular sections.

In general, in nonprismatic columns \(b = b(x)\) and \(h = h(x)\). If either \(b\) or \(h\) remains constant, we use the term “plane-uniformly tapered columns”. If they change proportionally to each other, we call them “spatially uniformly tapered columns” (Życzkowski 1955). In the general case they may be called “spatially nonuniformly tapered”, but they always belong to affine columns, since all rectangles are affine to each other (Gajewski and Życzkowski 1988). Nonprismatic columns of other cross-sectional shape may exceed affine columns, since then the transformation of the cross-section may exceed affinity. Here we consider only plane-tapered or spatially uniformly tapered columns.

The system of coordinates is shown in Fig. 1. The cross-sectional dimension \(h\) will always correspond to the axis \(y\), and the dimension \(b\) – to the axis \(z\), irrespective of the direction of buckling. First we consider the most general case of anisotropic stress corrosion, assuming that it takes place in both transversal directions \(y\) and \(z\), but with different intensity, for example due to different protection in each direction. Corrosion in the direction \(x\) (longitudinal) will not be considered. Hence we assume in both transversal directions the evolution equation (2) with \(\varphi(t) = \text{const.}\), but with different coefficients. We denote them by \(\alpha_y\) and \(\beta_y\) in the direction \(y\) (for the dimension \(h\)), and by \(\alpha_z\) and \(\beta_z\) in the direction \(z\) (for \(b\)). Allowing for corrosion from both sides of the section we obtain

\[
\frac{dh}{dt} = -2(\alpha_y + \beta_y \sigma) , \quad \frac{db}{dt} = -2(\alpha_z + \beta_z \sigma) ,
\]

where \(\sigma_t = \sigma = P/bh\), and compressive stresses are regarded as positive. First we eliminate \(\sigma\) from these equations and arrive at

\[
\frac{dh}{dt} = 2\frac{\alpha_y \beta_y - \alpha_y \beta_z}{\beta_z} + \frac{\beta_y}{\beta_z} \frac{db}{dt}. \quad (6)
\]

Equation (6) may readily be integrated. Suppose that loading \(P\) and corrosive environment are applied simultaneously at the moment \(t = 0\) and denote initial values of \(h\) and \(b\) by \(h_0\) and \(b_0\), respectively,

\[
h - h_0 = \frac{\alpha_y \beta_y - \alpha_y \beta_z}{\beta_z} t + \frac{\beta_y}{\beta_z} (b - b_0) . \quad (7)
\]

In view of (7) we may eliminate one of the unknowns from (5). Eliminating \(h\) and substituting into the second