Dislocation nucleation and work hardening in anti-plane constrained shear

1 Introduction

Plasticity in crystals and polycrystals is caused by nucleation, multiplication and motion of crystal defects. Defects appear in the crystal lattice to reduce its energy. Motion of defects yields the dissipation of energy, which, in turn, results in a resistance to the defect motion. The general structure of plasticity theory must, therefore, reflect this physical reality: an energy decrease by nucleation of defects and resistance to the defect motion due to dissipation. In classical plasticity theory these features are masked by the use of quite rough characteristics, the plastic strain, for the average description of dislocation network. In continuum theory of dislocations [1–3], these features are pronounced much better; in particular, the dislocation nucleation admits a clear characterization by the variational principle for finite plastic states [2].

This paper aims at studying the dislocation nucleation, the corresponding work hardening and the influence of resistance to the dislocation motion in an anti-plane constrained shear problem within the framework of continuum theory of dislocations. This problem admits an analytical solution and allows one to see clearly all the features mentioned. In particular, the solution obtained exhibits the energy and dissipation thresholds for dislocation nucleation, the Bauschinger translational work hardening, and the size effect.

The paper is organized as follows. In Sect. 2 the setting of the problem is outlined. Sections 3 studies dislocation nucleation at zero resistance by the energy minimization. In Sect. 4 the evolution of the dislocation network under the action of external shear and non-zero resistance is determined. Section 5 discusses the financial support by the DFG (German Science Foundation) within the Collaborative Research Center 526 (project D9) for K.C. Le is gratefully acknowledged

Communicated by L. Truskinovsky

V. L. Berdichevsky
Department of Mechanical Engineering, Wayne State University, Detroit, MI 48202, USA
E-mail: vberd@eng.wayne.edu

K. C. Le
Lehrstuhl für Allgemeine Mechanik, Ruhr-Universität Bochum, 44780 Bochum, Germany
the Bauschinger translational work hardening and the size effect. Finally, in Appendix the dual variational problem is constructed to show that the minimizer found in Sect. 3 is exact.

2 Anti-plane constrained shear

Consider the crystal beam undergoing an anti-plane shear deformation. Let $C$ be the cross section of the beam by planes $z = \text{const}$. For simplicity, we consider $C$ to be a rectangle of the width $a$ and height $h$, $0 < x \leq a$, $0 < y \leq h$. We place the crystal in a “hard” device with the prescribed displacement at the boundary $\partial C \times [0, L]$ (see Fig. 1)

$$w = \gamma y \quad \text{at} \quad \partial C \times [0, L],$$

where $w(x, y, z)$ is the $z$-component of the displacement and $\gamma$ corresponds to the overall shear strain. The hard device models the grain boundary. The height of the cross section, $h$, and the length of the beam, $L$, are assumed to be much larger than the width $a$ ($a \ll h, a \ll L$) to neglect the end effects and to have the stresses and strains depending only on one variable $x$ in the central part of the beam. If the shear strain is sufficiently small, then the crystal deforms elastically and $w = \gamma y$ everywhere in the specimen. If $\gamma$ exceeds some critical value, then the screw dislocations may appear. We allow only the slip planes parallel to the plane $y = 0$ and the dislocation lines parallel to the $z$-axis. Our aim is to determine the distribution of dislocations as function of $\gamma$ within the framework of continuum theory of dislocations proposed in [1–3]. For screw dislocations with the slip planes parallel to the plane $y = 0$, the tensor of plastic distortion, $\beta_{ij}$, has only one non-zero component $\beta_{zy} \equiv \beta$. We assume that $\beta$ depends only on $x$-coordinate: $\beta = \beta(x)$. Since the displacements are prescribed at the boundary of the crystal, dislocations cannot penetrate the boundaries $x = 0$ and $x = a$, therefore

$$\beta(0) = \beta(a) = 0. \quad (1)$$

The plastic strains are given by

$$\varepsilon^{(p)}_{yz} = \varepsilon^{(p)}_{zy} = \frac{1}{2} \beta(x).$$

The only non-zero component of the tensor of dislocation density, $\alpha_{ij} = \varepsilon_{jkl} \beta_{il,k}$, is

$$\alpha_{zz} = \beta_{,x},$$

where the comma in indices denotes the spatial derivative with respect to the corresponding coordinate. Energy of the crystal is a sum of the elastic energy of homogeneous deformation and the energy of dislocation network. The latter must depend linearly on the dislocation density for small densities and becomes infinite to avoid