Abstract The present paper is devoted to a model for elastic layered prismatic shells which is constructed by means of a suggested in the paper approach which essentially differs from the known approaches for constructing models of laminated structures. Using Vekua’s dimension reduction method after appropriate modifications, hierarchical models for elastic layered prismatic shells are constructed. We get coupled governing systems for the whole structure in the projection of the structure. The advantage of this model consists in the fact that we solve boundary value problems separately for each ply. In addition, beginning with the second ply, we use a solution of a boundary value problem of the preceding ply. We indicate ways of investigating boundary value problems for the governing systems. For the sake of simplicity, we consider the case of two plies, in the zeroth approximation. However, we also make remarks concerning the cases when either the number of plies is more than two or higher-order approximations (hierarchical models) should be applied. As an example, we consider a special case of deformation and solve the corresponding boundary value problem in the explicit form.

Keywords Layered shells · Cusped layered shells · Microtemperatures

1 Introduction

A vast literature is devoted to two-dimensional (2D) models of multilayered structures (layered plate and shell structures, sandwich structures) under the action of a mechanical field or its combination with other physical fields (thermal, electrical, magnetic). These 2D models can be seen either (i) in an equivalent single-layer form, when all the layers of the multistructure are considered as one equivalent structure, or (ii) in a layer-wise form, when each layer (ply) of the multistructure is separately considered in order to write the expansions in the thickness direction of displacements and stresses for each layer, or (iii) in a mixed form with displacements in the equivalent single-layer form and stresses in the layer-wise form.

The classical laminate theory is based on Kirchhoff–Love hypothesis [1–3] and belongs to the first type according to the above-mentioned classification. To the same type belongs, the first-order shear deformation theory which is the extension of the so-called Reissner–Mindlin model to multilayered structures [3,4]. A refinement of the last theory was made by applying Vlasov’s model for a single-layered plate which led to the
Vlasov-Reddy theory of the laminated plates \([4,6,7]\) (see also \([3,5]\)). Other equivalent single-layer theories are given in \([8–10]\).

In \([1]\) (see also \([3,11]\)), layer-wise theories of multilayered structures (plates, shells) are considered (see also the references given in \([1,3,11]\)).

Carrera’s unified formulation is a technique which handles a large variety of multilayered plates and shells in a unified manner \([12–14]\) (see also \([15]\)).

In these models, interface (interlaminar) conditions look like \([16,17]\):

- in-plane stresses may be discontinuous;
- transverse stresses must be continuous;
- displacements must be continuous in thickness direction for compatibility reasons;
- displacements may have discontinuous first derivatives.

A survey \([18]\) is devoted to asymptotic expansions and hierarchical models for plates and shells, including laminated ones (see also references given there).

The suggested in the present paper model actually belongs to the second type according to the above-stated classification. But in contrast to the above-mentioned engineering models: (i) for constructing hierarchical models for each, maybe cusped (tapered), layer \(k, k = \overline{1,n}\), of the elastic layered prismatic shell modified Vekua’s dimension reduction method is used which, roughly speaking, means expansion of displacements \(u_i, i = 1,2,3\), in the Fourier–Legendre series with respect to the thickness variable and keeping as their approximate values first \(N_i(1) + 1, i = 1,2,3\), in terms of expansions and neglecting other ones in the hierarchical model (approximation) of the order \(N^{(k)} := (N_1^{(k)}, N_2^{(k)}, N_3^{(k)})\) (see \([19]\)); (ii) layers are perfectly bonded, which will be fulfilled up to the accuracy of the models belonging to the hierarchical family; (iii) from the three-dimensional (3D) setting of a boundary value problem (BVP), the corresponding BVP for each layer is derived in such a way that for the first layer BVP can be solved independently and beginning with the second layer BVP for each layer can be solved independently using the solution of the BVP for the previous layer.

Modifying correspondingly differential hierarchical models constructed in \([20]\) for elastic prismatic shells of variable thickness with microtemperatures (for such materials, the thermodynamics is completed by formulating the second law of thermodynamics for a continuum with microtemperatures \([21]\)), we can construct a model, similar to the suggested in the present paper model, for layered prismatic shells consisting of elastic materials with inner structure whose particles, in addition to the classical displacement and temperature fields, possess microtemperatures.

Let a layered prismatic shell consist of \(n\) prismatic layers \(P_k, k = \overline{1,n}\), as plies with the upper \(x_3 = h(x_1, x_2), k = \overline{1,n}\), and lower \(x_3 = h(x_1, x_2), k = \overline{1,n}\), piecewise smooth face surfaces, correspondingly; herewith,

\[
h^{(+)}(x_1, x_2) = \Phi_{k-1}(x_1, x_2), \quad (x_1, x_2) \in \omega, \quad (1)
\]

and lateral boundaries \(S_k, k = \overline{1,n}\), where a domain \(\omega\) is the common for all the prismatic shells projection on the plane \(x_3 = 0\). \(S := \bigcup_{k=1}^{n} S_k\) denotes the joined lateral cylindrical surface with a generatrix parallel to the \(x_3\) axis according to the definition of prismatic shells (see \([22–24]\)).

Allowing cusped (tapered) edges for each ply, evidently, the thickness of the plies (see Fig. 1)

\[
2h(x_1, x_2) := h^{(+)}(x_1, x_2) - h^{(-)}(x_1, x_2) \geq 0, \quad k = \overline{1,n},
\]

and the thickness of the layered prismatic shell, by virtue of (1),

\[
2h(x_1, x_2) := \frac{h^{(+)}(x_1, x_2) - h^{(-)}(x_1, x_2)}{n} = \sum_{k=1}^{n} 2h_k \geq 0,
\]

where the equality can take place only on the boundary of \(\omega\).

Let

\[
x_{ijl}(x_1, x_2, t), \quad e_{ijl}(x_1, x_2, t), \quad u_{ijl}(x_1, x_2, t), \quad \Phi_{ijl}(x_1, x_2, t), \quad i, j = 1,2,3, \quad l = 0,1,2,\ldots, \quad k = \overline{1,n},
\]