Spiral and Wavy Vortex Flows in Short Counter-Rotating Taylor–Couette Cells

Olivier Czarny, Eric Serre, and Patrick Bontoux
LMSNM, FRE 2405 CNRS – Universités d’Aix-Marseille,
IMT, La Jetée-Technopôle de Château-Gombert,
38 rue Frédéric Joliot-Curie, 13451 Marseille cedex 20, France

Richard M. Lueptow
Department of Mechanical Engineering, Northwestern University,
Evanston, IL 60208, U.S.A.
r-lueptow@northwestern.edu

Communicated by H.J.S. Fernando
Received 5 November 2001 and accepted 29 March 2002
Published online 2 October 2002 – © Springer-Verlag 2002

Abstract. Differentially rotating cylinders result in a rich variety of vortical flows for cylindrical Couette flow. In this study we investigate the case of a short, finite-length cavity with counter-rotating cylinders via direct numerical simulation using a three-dimensional spectral method. We consider aspect ratios ranging from 5 to 6. Two complex flow regimes, wavy vortices and interpenetrating spirals, occur with similar appearance to those found experimentally for much larger aspect ratios. For wavy vortices the wave speed is similar to that found for counter-rotating systems and systems in which the outer cylinder is stationary. For the interpenetrating spiral structure, the vortices are largely confined to the unstable region near the inner cylinder. The endwalls appear to damp and stabilize the flow as the aspect ratio is reduced to the point that in some cases the vortical flow is suppressed. At higher inner cylinder speeds, the interpenetrating spirals acquire a waviness and the vortices, while generally near the inner cylinder, can extend all of the way to the outer cylinder.

1. Introduction

The Taylor–Couette system, consisting of shear flow between differentially rotating concentric cylinders, provides valuable insight into the stability of flows in rotating systems and the interaction of various vortical structures. The Taylor–Couette configuration with a fixed outer cylinder has been investigated experimentally and numerically in great detail since Taylor's pioneering work (Taylor, 1923). Experiments for infinitely long cylinders have established that as the rotational speed of the inner cylinder increases, the flow changes according to the following scenario: stable Couette flow, axisymmetric Taylor vortices, wavy vortices, modulated wavy vortices, and turbulent vortices (Andereck et al., 1986; Fenstermacher et al., 1979). The velocity field in non-wavy and wavy vortex has been studied in detail experimentally (Akonur and Lueptow, 2002; Wereley and Lueptow, 1998) and numerically (Coughlin and Marcus, 1992a,b; Marcus, 1984). However, the
flow state for wavy vortex flow depends strongly on how the final rotational speed is achieved (impulsively or quasi-statically) (Coles, 1965; Ghoshmoulic and Yao, 1996).

The rotation of the outer cylinder in addition to the inner cylinder results in a variety of other flow regimes for long cylinders: wavy inflow and outflow, wavelets, twisted vortices, and corkscrew regimes for co-rotating cylinders; interpenetrating spirals, wavy interpenetrating spirals, intermittent turbulent spots, and spiral turbulence regimes for counter-rotating cylinders (Andereck et al., 1986). Several of these flow regimes, particularly spiral turbulence, have been studied experimentally in long cylinders (Coles, 1965; Colovas and Andereck, 1997; Edwards et al., 1991b; Goharzadeh and Mutabazi, 2001; Hegseth et al., 1989, 1996; Snyder, 1970; Vanatta, 1966). Computational studies of differentially rotating cylinders have considered both the co-rotating case (Antonijuan and Sanchez, 2000) and counter-rotating case (Antonijuan et al., 1998; Jones, 1982; Sanchez et al., 1993), without any axial confinement. In addition, the chaotic nature of the flow between long counter-rotating cylinders has been studied using bifurcation theory (Chossat and Iooss, 1994; Golubitsky and Langford, 1988; Stern and Hussain, 1994).

Experimentally, the endwalls strongly influence the flow by constraining the axial motion of the flow and by pumping fluid in the endwall boundary layers. Away from the endwalls the flow is geostrophic, so that the centrifugal force due to the azimuthal velocity is balanced by the pressure gradient force resulting in no radial flow. This balance is upset near the endwalls where the no-slip boundary condition results in an azimuthal velocity near the endwall that is different from that away from the endwall. The imbalance between the pressure gradient imposed away from the endwall and the centrifugal force related to the azimuthal velocity in the endwall boundary layer results in a radial flow near the endwall. For the case of the inner cylinder rotating with the outer cylinder fixed, the direction of the radial flow in the Ekman layer is easy to predict. For endwalls rotating with the inner cylinder, centrifugal viscous pumping causes an outflow at the endwalls. For endwalls fixed to the outer cylinder, the imbalanced pressure gradient and centrifugal forces result in an inflow at the endwalls. For differentially rotating cylinders, the situation is more complicated. Depending on the rates and directions of rotation of the cylinders and the endwalls, the force imbalance can result in inflow, outflow, or both (depending on radial position). The direction of the radial flow near the endwalls can be predicted by comparing the pressure gradient force away from the endwalls with the centrifugal force at the endwall. The imbalance in these forces drives the flow at the endwalls. This Ekman layer flow at the endwalls determines the rotation of the vortices nearest the endwalls, which appear well below the critical speed for Taylor vortices (Andereck et al., 1986; Koga and Koschmieder, 1989; Sobolik et al., 2000). These endwall vortices subsequently determine the rotation of the entire vortex structure above the critical speed for Taylor vortices, even for relatively long cylinders (Burkhalter and Koschmieder, 1973).

For short cylinders with the outer cylinder fixed, the endwalls play a significant role: the transition to non-wavy vortical flow occurs at a lower rotational speed and the subsequent transition to wavy vortical flow occurs at a higher rotational speed than in the case of infinitely long cylinders because the endwall vortices related to the Ekman layers alter the vortical structure (Cole, 1976). Furthermore, the axial wave number, which is such that pairs of nearly square vortical cells appear in the infinite cylinder case, can be strongly influenced by the distance between the endwalls. The key parameter is the aspect ratio, \( L = 2h/d \), where \( 2h \) is the distance between the endwalls and \( d \) is the annular gap. For aspect ratios that are even integers, the axial wavelength of the vortex pair is that predicted by theory for identical endwall conditions. However, an odd or non-integer aspect ratio results in vortices, particularly those near the endwalls, stretching or compressing to accommodate the proper vortical rotation defined by the endwall conditions (Cole, 1976). In some cases, “anomalous” vortex structures have been identified where the rotation of the endwall vortices is opposite that predicted based on endwall conditions (Cliffe and Mullin, 1985).

The effect of endwall conditions on the flow for differently rotating cylinders was of concern in the classic paper by Andereck et al. (1986), even though the aspect ratio was \( L = 30 \) to minimize endwall effects. In this paper we address the flow between short counter-rotating cylinders in which the Ekman layers near the endwalls interact with the vortical structures. To our knowledge, there have been no investigations, experimental or computational, of endwall effects in short differentially rotating cylindrical Couette systems. We use direct numerical simulation based on a spectral Chebyshev–Fourier method that has been shown...