Asymptotic solutions of the axisymmetric moist Hadley circulation in a model with two vertical modes

Abstract A simplified model of the moist axisymmetric Hadley circulation is examined in the asymptotic limit in which surface drag is strong and the meridional wind is weak compared to the zonal wind. Our model consists of the quasi-equilibrium tropical circulation model (QTCM) equations on an axisymmetric aquaplanet equatorial beta-plane. This model includes two vertical momentum modes, one baroclinic and one barotropic. Prior studies use either continuous stratification, or a shallow water system best viewed as representing the upper troposphere. The analysis here focuses on the interaction of the baroclinic and barotropic modes, and the way in which this interaction allows the constraints on the circulation known from the fully stratified case to be satisfied in an approximate way. The dry equations, with temperature forced by Newtonian relaxation towards a prescribed radiative equilibrium, are solved first. To leading order, the resulting circulation has a zonal wind profile corresponding to uniform angular momentum at a level near the tropopause, and zero zonal surface wind, owing to the cancelation of the barotropic and baroclinic modes there. The weak surface winds are calculated from the first-order corrections. The broad features of these solutions are similar to those obtained in previous studies of the dry Hadley circulation. The moist equations are solved next, with a fixed sea surface temperature at the lower boundary and simple parameterizations of surface fluxes, deep convection, and radiative transfer. The solutions yield the structure of the barotropic and baroclinic winds, as well as the temperature and moisture fields. In addition, we derive expressions for the width and strength of the equatorial precipitating region (ITCZ) and the width of the entire Hadley circulation. The ITCZ width is on the order of a few degrees in the absence of any horizontal diffusion and is relatively insensitive to parameter variations.

Keywords Hadley circulation · Tropical dynamics · Geophysical fluid dynamics

1 Introduction

The Hadley circulation is the axisymmetric component of the meridional (north–south) overturning of the Earth’s tropical atmosphere. That is, it is the component of that overturning which results when the flow field
is averaged in the zonal (east–west) direction. In the annual mean, air rises near the equator, accompanied by copious rainfall as the moisture contained in the warm near-surface air condenses at lower temperatures higher up. This, now dehydrated, upper-level air then flows poleward, and descends at subtropical latitudes, where little rain occurs since the air is dry. The upper-level poleward flow also transports absolute angular momentum, which the equatorial atmosphere generally has in excess at higher-latitude regions due to the rotation of the Earth and the greater distance from the Earth’s axis at the equator. This leads to large relative angular momentum, or strong upper-level westerly (eastward) flow relative to the surface, in the subtropical jets at the poleward edges of the circulation. At the edge of the jet the air descends to the surface, where it flows equatorward, losing angular momentum to surface drag. Surface fluxes restore both the angular momentum and humidity of near-surface parcels towards those of the surface, until they ascend again near the equator; the angular-momentum exchange leads to the existence of surface easterlies near the equator and surface westerlies further poleward. These are arguably the most fundamental first-order features of the general circulation of the atmosphere, which dynamical meteorology should be able to explain.

Superimposed on the zonal mean circulation are eddies, or nonaxisymmetric flow features that disappear in the zonal average. These eddies can be quite large, particularly in the extratropics but also in the tropics, and many important features of the general circulation are undeniably related to these eddies. The axisymmetric Hadley circulation is also influenced by eddies, as the flow is nonlinear and eddy transports (rectified wave–wave interaction terms in the zonally averaged equations of motion) can be important to the zonal mean flow. Despite this, axisymmetric models of the Hadley circulation, in which no eddy effects are considered, have been quite useful in building our understanding of the general circulation. Even though there is now increasing evidence that eddies are even more important to the Hadley circulation than previously recognized [25], axisymmetric models will remain a valid starting point. The eddies exist in the first place because the flow which would occur in their absence is baroclinically unstable. Thus studying the axisymmetric circulation is a necessary prerequisite to understanding the complete circulation [18,19].

The vertical structure in studies of the axisymmetric Hadley circulation is of two types. Some studies consider a fully stratified atmosphere, in which the interior of the atmosphere is considered to be nearly inviscid, as described in Schneider [19] and further developed by Held and Hou [7]. This leads to conservation, and homogenization of angular momentum in the interior, and thus to very (in fact unrealistically) strong subtropical jets. Surface drag, however, is essential to the circulation as it restores the angular momentum of near-surface air parcels back to that of the Earth’s surface, thus setting the value of the angular momentum for the entire circulation. Because the interior is nearly inviscid, the effects of surface drag are felt only in a thin boundary layer near the surface in these fully stratified models. Other studies consider only a single layer of fluid, obeying a form of the shallow water equations, best thought of as representing only the inviscid upper troposphere [8,9,16,21]. A flow-dependent mass source appears in the equation for the depth of the layer, representing mass transport between the modeled layer and the lower troposphere. In these calculations, surface drag enters implicitly through the value of the angular momentum (or zonal velocity) imputed to the lower-tropospheric mass source that forces the modeled layer.

In contrast to Hadley cell theory, the theory for most other features of the tropical atmospheric circulation favors a spectral, or modal representation of the vertical structure. This is not formally justifiable due to the lack of a discrete upper boundary on the atmosphere (in contrast to the ocean), which prevents the existence of normal-mode solutions, but nonetheless has been shown to give at least qualitatively acceptable results in many circumstances. In many theories, starting from Matsuno [12], only the so-called first baroclinic mode, containing horizontal velocity maxima of opposite sign at the surface and the tropopause with one zero crossing between, is retained. This leads to a set of shallow water equations, but with a different interpretation than that typically used in Hadley cell theory. Rather than representing flow in layers and neglecting vertical structure within each layer, the equations represent the entire tropospheric flow and assume a specific nontrivial form for the vertical structure. Because the first baroclinic mode velocity has a maximum at the surface, it is incapable of representing the interaction of the flow with the surface drag, which acts to bring the near-surface flow to zero without directly affecting upper levels. As shown below, in order to achieve a plausible representation of the interaction with the surface drag, a barotropic or external mode must be added, with a uniform (or at least uniform in sign) velocity throughout the troposphere. In a model with one barotropic and one baroclinic mode, the two modes can cancel at the surface while adding constructively at upper levels, yielding small surface winds and strong upper level winds, in qualitative agreement with observations.

In this study, we analyze the axisymmetric Hadley circulation in a two-mode model of this type, namely the quasi-equilibrium tropical circulation model (QTCM; 13). Our first goal is to make the connection between existing axisymmetric Hadley cell theory, which does not use modal decompositions, and the theory for the