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Oded Regev (2006): Chaos and Complexity in Astrophysics
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The theory of nonlinear dynamical systems, commonly referred to as ‘chaos theory’, had its origin in celestial mechanics but has applications to fluid dynamics and plasma physics as well as to astronomy and astrophysics. At the end of the nineteenth century, Poincaré had already recognized the presence of Hamiltonian chaos in solutions of the three-body problem, but the subject lay dormant for another 50 years. Electrical engineers were aware of aperiodic behaviour but mathematicians believed that solutions of nonlinear differential equations were quasiperiodic (i.e. multiply periodic) until Cartwright and Littlewood demonstrated the presence of behaviour that we would now call chaotic in solutions of the van der Pol equation. Unfortunately, their paper was so obscure that there were only a few islands of comprehension scattered around the mathematical world. It was not until some time after the appearance in 1963 of Lorenz’s epoch-making paper and Smale’s horseshoe proof that the notion of chaotic behaviour, with sensitive dependence on initial conditions, came to be generally accepted. Now the ‘butterfly effect’ has entered common parlance, and books on chaos theory abound.

This volume differs from most of them in being directed primarily at astrophysicists. That said, it can really be regarded as two distinct parts. The first, occupying almost 60% of the text, is a general introduction to the nonlinear dynamics of Hamiltonian and dissipative systems, which could form the basis of a final year or graduate course for a more general audience. The second part, on the other hand, is concerned with a selection of specifically astrophysical applications, drawn from celestial mechanics, stellar pulsation, the formation of complex patterns and astrophysical fluid dynamics.

The first part begins with a few illustrative examples, slanted towards an astrophysical audience. These naturally start with the logistic map, followed by Baker’s simplified one-zone model of stellar pulsation (leading to a forced Duffing equation), regular and chaotic behaviour in a Hénon-Heiles potential, the Moore–Spiegel example of chaotic behaviour in a simple model of overstable convection, and an introduction to the formation of localised fronts in a thermally unstable medium. These examples motivate the possibly bewildered student to embark upon a systematic study of nonlinear systems. This focuses on differential equations (or ‘flows’), with occasional cross-references to maps. It begins with a standard classification of fixed points, followed by a discussion of periodic, homoclinic and heteroclinic orbits, and then (perhaps prematurely) of Poincaré sections and Liapunov exponents. The treatment next moves on to bifurcation theory, with a classification of codimension-one bifurcations and an introduction to centre-manifold reduction, before switching to introduce fractal sets and various types of fractal dimension.

Having absorbed all this material, the student is ready for an informal introduction to chaotic dynamics, beginning with dissipative systems and proceeding by way of examples. Sensitive dependence on initial conditions is introduced via the horseshoe map, and period-doubling is described for the logistic map, while the circle map is brought in to illustrate the quasiperiodic route to chaos (though, as Arnold has pointed out, the latter has “an exotic character”). After a brief account of intermittency, as a third route to chaos, we return to homoclinic and heteroclinic orbits in two-dimensional Poincaré maps (now regarded as derived from flows)

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and consider transversal intersections, leading to homoclinic and heteroclinic tangles and the appearance of horseshoes. There is also a brief mention of Shilnikov’s route to chaos near a homoclinic connection to a saddle-focus in a third-order flow.

The text now jumps to consideration of time series, power spectra and embedding procedures, which leads naturally to a further discussion of fractal dimensions. The next jump takes us to Hamiltonian systems, with a brief summary of Hamiltonian dynamics. The description of integrable systems, with motion on tori in phase space, is introduced in terms of action-angle variables. The treatment then goes on to consider nearly integrable systems, using a slightly modified Hénon-Heiles potential as an example, and reveals the small divisors problem. This is followed by an (inevitably abbreviated) account of KAM theory, with reference to the standard twist map. A distinction is drawn between this form of ‘soft chaos’, where some KAM tori still persist (though Arnold diffusion occurs in systems with more than two degrees of freedom), and ‘strongly irregular motion’ with mixing over the entire phase space. This chapter succeeds in providing a clear but brisk introduction to Hamiltonian chaos.

The final chapter of the first part is devoted to spatially extended systems: these are described by nonlinear partial differential equations and can display a variety of spatial patterns. A distinction is made between cellular and extended behaviour; the former gives rise to stationary patterns (whose evolution may be described, for example, by a Landau equation), while the latter allows travelling patterns, whose overall behaviour may be represented by an envelope equation (for instance, the complex Ginzburg–Landau equation). The chapter concludes with a discussion of nonlinear waves and the generation of fronts and solitons in the Burgers and Korteweg–de Vries equations, followed by a more general discussion of multifront defects.

Taken all in all, this first part of the book provides a good introduction to nonlinear systems, slanted towards the interests of astrophysicists. Internal evidence suggests that the text must have grown out of courses given over a number of years. There are copious references, though few of them date from later than 1995. Among obvious omissions, I noted the books by Glendinning, Kuznetsov and Wiggins, and, more seriously, Arnold’s

The second part has a very different flavour, for it focuses on a limited number of specific applications of dynamical systems theory to problems in astronomy and astrophysics. The success of these applications is closely related to the availability of computational techniques, with access to ever more powerful computers. Professor Regev introduces his account with a spirited defence of the use of analytical techniques in order to understand the basic physics of astronomical objects: “In a growing part of astrophysical literature the adjective ‘nonlinear’ is often a shorthand for ‘that must be calculated numerically’. This is in my view a symptom of the trendy ‘common knowledge’ that a full scale, brute-force computer simulation is the ultimate goal of a theoretical astrophysicist.” I heartily agree. Intelligent treatment of nonlinear problems requires a sophisticated combination of theory and numerics.

The second obvious application are the $n$-body problem for mutually attracting point masses, and nonlinear stellar oscillations, the first as the classic example of Hamiltonian chaos and the second as a dissipative system. The chapter on planetary, stellar and galactic dynamics naturally begins with Poincaré’s study of the reduced three-body problem and goes on to review the Sun–Earth–Moon system. A key question in such a (non-relativistic) system is whether it is stable, or whether the lightest body can escape—or even achieve a singular escape, reaching an infinite velocity in a finite time. Further complications arise from spin–orbit coupling when tidal forces are taken into account, as in the chaotic tumbling of Saturn’s moon Hyperion. Resonances and chaotic behaviour also affect the orbits of asteroids and dwarf planets such as Pluto. On a larger scale, chaos affects the formation of binary stars by tidal capture within a globular cluster as well as the dynamics of galactic orbits. The Hamiltonian treatments covered in this chapter, all of which rely on careful combinations of mathematical theory with extremely precise numerical computation, provide the best examples of chaos in astrophysics—but a detailed account would require a book in itself, such as that by Murray and Dermott.

Pulsating stars offer a promising field for applications of nonlinear dynamics. Photometric measurements of light curves for many stars have been compiled for decades, and Buchler and his colleagues have analysed such time series for several variable stars. From these data they find clear evidence of low-dimensional behaviour, with a pair of nonlinearly coupled modes that are close to 2:1 resonance. This accords with theoretical expectations: simple toy models, such as that devised by combining Baker’s one-zone model of stellar oscillations with Moore and Spiegel’s nonlinear oscillator, can be generalised to describe transitions to chaos, for instance via period-doubling or by Shilnikov’s mechanism.