Absolute and Convective Instabilities in the Compressible Boundary Layer on a Rotating Disk

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Abstract. A numerical study has been undertaken to investigate the nature of inviscid instability of the three-dimensional compressible boundary layer flow due to a rotating disk. The compressible Rayleigh equation is integrated using a spectral Chebyshev-collocation method together with a fourth-order Runge–Kutta integrator. In the context of spatio-temporal stability analysis, the singularities of the resulting dispersion relation are determined and the ones that satisfy the Briggs–Bers pinching criterion have been selected. In certain finite parameter regions of eigenvalues (wave numbers and wave angles, for instance) it is found that by varying the Mach number, absolute instability occurs in the compressible boundary layer on a rotating disk. The range corresponding to the incompressible flow case given in Lingwood (1995) (π between 14.615° and 38.114°) is verified. The results of Cole (1995) are also verified. The overall effect of compressibility is to reduce the extent of absolute instability at higher Mach numbers. The effect of heating the wall is to enhance the absolute instability properties, however, cooling the wall is found to decrease greatly the region of absolute instability regime for the range of Mach numbers studied. It is also shown in this study that for non-insulated walls a direct spatial resonance of the eigenmodes is possible and this raises the possibility of large local algebraic growth of perturbations being important in some instances.

1. Introduction

The development of instability waves and the mechanisms through which they lead to transition to turbulence in three-dimensional boundary layer flows are of fundamental importance in hydrodynamic stability theory.

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Recently there has been renewed interest in whether such flows are absolutely or convectively unstable. The absolute or convective nature of a linearly unstable flow determines how a disturbance, introduced into the boundary layer and initially small, will evolve in time and space. Temporal stability theory assumes that a disturbance will evolve in time from some initial spatial distribution, and so the complex frequency has to be determined as a function of the real wave number. In spatial stability theory it is assumed that a disturbance will evolve as it moves downstream, that is the complex wave number has to be determined as a function of the real frequency. An unstable flow is classified as being absolutely unstable if its response (expressed by the Green’s function, see Huerre and Monkewitz (1985) for instance) to an impulse in time and space amplifies unboundedly everywhere in space for large time. If, on the other hand, the impulse response at a fixed location in space grows and then ultimately decays after some time, then the flow is classified as being convectively unstable. Therefore, with the above considerations, in absolutely unstable flow any transient disturbance will grow exponentially large everywhere (the assumptions of linear stability will of course break down at some point and nonlinear effects will become important) contaminating the flow upstream and downstream of its point of origination. However, in convectively unstable flow a transient disturbance will be convected away as it grows, eventually leaving the basic flow undisturbed. From the above it can be seen that spatial stability theory is only appropriate if the flow is convectively unstable. For absolute instability on the other hand a combined spatio-temporal analysis is required.

The concept of absolute/convective instabilities has been extensively studied in the field of plasma physics, see, for example, Briggs (1964), Bers (1975) and for a review of absolute/convective instabilities in plasma physics see Bers (1983). Although these investigations are concerned primarily with the instability in plasmas, the absolute/convective classification of an instability is an important issue with regard to flow control in applications in fluid mechanics. If the flow is convectively unstable, then a localized forcing in order to control spatial development of disturbances will be effective. A self-excited growing instability in an absolutely unstable flow, on the other hand, will overwhelm the forcing. Therefore, it is extremely useful to define the parameter range wherein the flow is either absolutely or convectively unstable. The recent work of Ceng et al. (1997) on the viscous liquid curtain problem demonstrates the importance of identifying such regions. For a comprehensive review of absolute/convective instabilities which in particular lead to self-excited global modes in fluid flows, see Huerre and Monkewitz (1990).

The concepts of absolute/convective instabilities have also been applied to mixing-layer velocity profiles. Huerre and Monkewitz (1985) using a tanh velocity profile have shown how the flow becomes convectively unstable for values of the velocity ratio less than 1.315, otherwise it is absolutely unstable. This result differs from the intuitively expected result of the flow being absolutely unstable for values of the velocity ratio greater than unity, when a counterflow first appears. Most experimental studies of mixing-layer flow are performed with a velocity ratio less than one, hence they are convectively unstable and spatial stability theory should be used in any theoretical comparison. Pavithran and Redekopp (1989) and Jackson and Grosch (1990) have extended the analysis of Huerre and Monkewitz (1985) to include the effects of compressibility. They found that as the Mach number is increased the velocity ratio needed to produce absolute instability also increases. For fixed values of the Mach number, cooling the low-speed stream relative to the high-speed stream extends the domain of absolute instability. For sufficiently large cooling it was found that an absolute instability could occur without any counterflow.

Three-dimensional boundary layers are susceptible to a strong inviscid instability, known as cross-flow instability, because of the inflexional character of the flow in certain flow directions, see, for example, Gregory et al. (1955) and the review by Reed and Saric (1989). The flow due to a rotating disk has been extensively used as a prototype for studying the instability of other fully three-dimensional boundary layer flows, such as the flow over swept wings, and linear stability calculations have been carried out by Malik (1986), Balakumar and Malik (1990), among others. Balakumar and Malik (1990) present neutral and growth rate curves for various frequencies including those for the stationary disturbances. The disk flow has also been studied experimentally by several people including Gregory et al. (1955), Kobayashi et al. (1980), Wilkinson and Malik (1983), and these and other experiments are described in the review paper by Reed and Saric (1989).

In this investigation we are concerned with finding the absolute/convective nature of the inviscid instability in the flow due to a rotating disk. For values of the Reynolds number in the range 200–500, an experimental study by Wilkinson and Malik (1985) showed that disturbances propagate and grow as wave packets from isolated roughness elements on the surface of a rotating disk. Mack (1985) calculated numerically the