Compositional synthesis of optical imaging systems

Devika Subramanian, Ron Goldman

Abstract
This paper introduces a new systematic and complete compositional technique for conceptual design of optical imaging systems from behavioral specifications. There are two key ideas: (1) modeling the structure and behavior of optical components using affine transformations, and (2) using a systematic search algorithm integrated with a powerful algebraic constraint solver to compute three-dimensional layouts of optical components in the design. By embedding our tool in a constraint programming environment, we are able to automate the conceptual design of optical imaging systems – a step performed manually by human designers until now. We demonstrate the power of the method by recreating several designs for imaging systems of copiers.

Keywords
Conceptual design, Optical system synthesis, Constraint representations, Constraint solving, Affine transformations

1 Introduction
The goal of our research is to derive computational theories of conceptual or preparametric design for optical imaging systems. Examples of such systems include VCRs, optical gyroscopes, and imaging systems of cameras and copiers. Optical imaging system synthesis solves the problem of determining the three-dimensional structure of a device composed of optical components that implements a given behavioral specification. Optical and optomechanical systems are ubiquitous today; yet there are few programs that can automatically design and simulate them at a conceptual or preparametric level.

The most critical design decisions are made in the preparametric design stage: viz. conceptual studies and layout design [29]. Errors made at this level are very costly to correct at the later stages. Conceptual design is difficult because a variety of task-specific constraints ranging from geometry, kinematics, dynamics, materials, assembly, and costs need to be considered simultaneously. The standard method for generating conceptual designs is to hold brainstorming sessions with human experts from each of these areas to propose, check, and satisfy constraints from their individual domains. This process is slow, nonsystematic, uncoordinated, and error-prone since it leaves no audit trail of key design decisions. It can lead to disasters when people overlook important constraints or when not all feasible alternatives are considered.

This paper proposes a design tool for automatic generation of conceptual alternatives, i.e. high-level descriptions of the topology of a device. We distinguish between preparametric conceptual design and parametric design. Conceptual design generates device configurations and establishes constraints on key design parameters. Parametric design involves finding optimal values of parameters for a given configuration. The key idea in our paper is to construct design topologies by decomposing given behavioral specifications into primitive design fragments, which can then be implemented using available components. The specification of desired behaviors is in the form of sets of constraints on object and image configurations, and the problem of generating alternative conceptual layouts is formulated as constraint satisfaction. Embedding the design tool in a constraint programming environment enables rapid generation of conceptual alternatives, and of testing designs by simulation to verify behavior. Underlying the design algorithms is a theory of compositional synthesis grounded in the following facts: (1) the behavior of optical components can be expressed as affine transformations on object and image configurations, (2) the overall input–output behavioral specification of the system itself can be expressed as an affine transformation on object and image configurations, and (3) a powerful algebraic constraint solver can systematically compose component behaviors symbolically and check if they implement the desired system behavior.

We illustrate our design system with an example. Consider the synthesis of an imaging system for a photocopier, where the task is to project an image of a stationary document page on to an imaging drum. To specify this design problem, we describe the position, orientation, and scale of the image with reference to the object. We treat the image as a scaled, rotated, and translated version of the object.
This is a partial specification of the device that captures its optical input–output behavior. The object is at the origin; the real image, which is the same size as the object (unity scaling along \(x\), \(y\), and \(z\)-axes with scaling origin \((0,0,0)\)), is at \((1,1,0)\) rotated by \(\pi\) about the \(z\)-axis with respect to the object orientation. We can also specify constraints on the volume or area of the overall device, or on the total optical path length. For this example, however, we focus on the input–output transformation alone, and compute a set of layouts of the optical components that achieves the given behavior.

The design generated by our system, consisting of a single lens and a pair of mirrors, is shown in Fig. 1. In fact, our program outputs a family of designs parameterized by \(x\), which is the shared \(x\)-coordinate of the mirrors and lens. The parameter \(x\) is in the open interval \((0,1)\). This is the smallest design in the sense of the fewest number of components needed to achieve the optical behavior. This design family has the principal lens axis parallel to the \(y\)-axis. When we orient the principal axis of the lens parallel to \((-1/\sqrt{2}, 1/\sqrt{2}, 0)\), we obtain the design shown in Fig. 2.

There are four unique aspects of our method. First, our layout synthesis algorithm is grounded in the laws of optics with optical components represented as linear transformations on object and image configurations. Second, our algorithm is complete; it enumerates all feasible design families that implement a given specification. Since our algorithm explores designs in increasing order of complexity as specified by the number of primitive components, we can guarantee minimality for the designs produced. For this example specification, there are no other designs involving fewer mirrors and lenses. Third, our algorithm computes key constraints on positions and orientations for all components in the design. These can then be instantiated for input into a detailed design program for further optimization and analysis. Our approach essentially allows for rapid generation of conceptual design sketches that can be refined with further computation. Fourth, even though we work from first principles in optics, we can obtain rapid generation of alternative design topologies. Our synthesis algorithm is fully implemented in Mathematica, and all of the examples presented in this paper are the output of our design system rendered using the Optica application in Mathematica. Our system has produced interesting designs for imaging systems for copiers as shown in Sect. 5.

This paper is organized as follows. In Sect. 2 we define object and image configurations and introduce transformations that map object to image configurations. We represent input–output specifications of optical imaging systems as affine transformations, viz. products of translation, rotation and scaling operations. We formulate the behavior of devices like mirrors, lenses and prisms (treated as special mirrors) as affine transformations. The operations that compose specifications and their concrete implementations are introduced at the end of Sect. 2. The foundations of a compositional theory of synthesis are described in Sect. 3. The general algebraic synthesis technique based on symbolic generation and testing of compositions of optical components is presented in Sect. 4. We demonstrate that the problem of designing layouts from specifications of object and image configurations is under-constrained. To overcome this problem, we develop a systematic search algorithm that refines problem specifications by adding three generic constraints, including one on the orientation of the principal optical axis of the system. In Sect. 5 we present computational results obtained with our framework in the domain of imaging systems for copiers. We conclude with a discussion of related work and avenues for future research in Sect. 6.

2 Specifying the behavior of optical imaging systems

We specify optical systems by their input–output optical behavior, i.e. the position, orientation, and scale of the

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\begin{align*}
\text{Input:} & \quad \begin{cases}
\text{obj pos} & (0,0,0) \\
\text{obj orient} & (1,0,0), (0,1,0), (0,0,1) \\
\text{img pos} & (1,1,0) \\
\text{img orient} & ((-1,0,0), (0,-1,0), (0,0,1)) \\
\text{img scale} & ((0,0,0), ((1,0,0), (1,1,0), (1,0,1)), (1,0,1))) \\
\text{real?} & +
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\text{Output:} & \quad \begin{cases}
\text{obj pos} & (0,0,0) \\
\text{obj orient} & (1,0,0), (0,1,0), (0,0,1) \\
\text{img pos} & (1,1,0) \\
\text{img orient} & ((-1,0,0), (0,-1,0), (0,0,1)) \\
\text{img scale} & ((0,0,0), ((1,0,0), (1,1,0), (1,0,1)), (1,0,1))) \\
\text{real?} & +
\end{cases}
\end{align*}
\]

Fig. 1a, b. An imaging system designed by our synthesis program. The top view and 3D arrangement of components are shown. The system transforms an object at \((0,0,0)\) into a real image of the same size at \((1,1,0)\) rotated by \(\pi\) about the \(z\)-axis with respect to the object.