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In fixture design, a workpiece is required to remain stable throughout the fixturing and machining processes in order to achieve safety and machining accuracy. This requirement is verified by a function of the computer-aided fixture design verification (CAFDV) system. This paper presents the methodologies of fixturing stability analysis in CAFDV. A kinetic fixture model is created to formulate the stability problem, and a fixture stiffness matrix (FSM) is derived to solve the problem. This approach not only verifies fixturing stability, but also finds the minimum clamping forces, fixture deformation, and fixture reaction forces. The clamping sequence can also be verified with this approach.

Keywords: Computer-aided fixture design verification (CAFDV); Stability analysis

1. Introduction

In fixture design, a workpiece is required to remain stable during the locating, clamping and machining processes in order to achieve safety and machining accuracy. The stability analysis module of the computer fixture design verification (CAFDV) is to verify the stabilities under these situations. These stabilities cannot be verified easily when the friction forces are taken into consideration. To achieve this, a kinetic fixture model is created to formulate the stability problem mathematically, and a fixture stiffness matrix (FSM) is derived to solve the problem.

With the kinetic fixture model, some other related problems can also be solved. The first is to find the minimum clamping forces required for stabilising the workpiece during the machining process, the second is to find the fixture deformation and its impact on machining accuracy, and the third is to find the influence of the clamping sequence on fixturing stability.

Many papers can be found on stability analysis, in both the fixturing and robot grasping areas. There are many different assumptions, approaches, and applications for stability analysis, such as the consideration of friction force, workpiece and fixture deformation, and clamping sequence. Table 1 shows the foci of related work.

The rest of the paper is organised as follows. First, the kinetic fixture model is introduced to formulate the stability problem. Then, several applications of the model are explored, including stability criteria, minimum clamping forces, clamping sequence, and fixture deformation. At the end, a case study and conclusions are given.

2. Kinetic Fixture Model

When a workpiece remains stable during the machining process (Fig. 1), it is balanced by two types of forces – external forces and internal forces. The external forces are active forces including clamping forces and machining forces, and the internal forces are reactive forces including reaction forces (with friction) from locators.

To simplify the problem, fixture components are assumed to be linear elastic bodies, and the workpiece is assumed to be a rigid body. It is noted that the integration with workpiece deformation is a separate task and is not in the scope of this study. When under external forces, locators may deform, and the workpiece may be displaced. In a stable situation, the workpiece may eventually be balanced between the external forces and the reaction forces generated from locator deformations.

To establish the workpiece equilibrium equation, here we use the wrench from screw theory [12] to represent the force and moment generated at a point. A wrench in 3D space \( \mathbf{W} = [F_x, F_y, F_z, M_x, M_y, M_z]^T \) consists of three force elements and three torque elements.

The workpiece is stable when the total external moment is balanced by the total internal moment, which is generated by the fixture reaction forces due to workpiece displacement and fixture deformation. This equilibrium equation is:
Table 1. Literature on fixturing stability.

<table>
<thead>
<tr>
<th>Study</th>
<th>Friction force</th>
<th>Minimum clamping force</th>
<th>Clamping sequence</th>
<th>Workpiece deformation</th>
<th>Fixture deformation</th>
<th>FEA method</th>
<th>optimisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chou et al. 1989 [1]</td>
<td>–</td>
<td>X</td>
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<td>Lee and Cutkosky, 1991 [2]</td>
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<td>Cogun, 1992 [3]</td>
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<td>X</td>
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<td>Xiong and Xiong, 1998 [5]</td>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>X</td>
<td>–</td>
</tr>
<tr>
<td>Rong et al. 1994 [6]</td>
<td>X</td>
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<td>Chen, 1995 [7]</td>
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<td>X</td>
<td>X</td>
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<td>King and Ling, 1995 [8]</td>
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<td>Wu et al. 1995 [9]</td>
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<tr>
<td>DeMeter, 1998 [10]</td>
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</table>

Fig. 1. Kinetic fixture model.

\[
\{W_i\} + \{W_e\} = [K] \cdot \{\Delta q\} + \{W_r\} = 0 \tag{1}
\]

or

\[
[K] \cdot \{\Delta q\} = -\{W_r\} \tag{2}
\]

where \(\{\Delta q\} = [\Delta x \ \Delta y \ \Delta z \ \Delta \alpha \ \Delta \beta \ \Delta \gamma]^T\) is the workpiece displacement,

\[
\{W_i\} = [F_{ix} F_{iy} F_{iz} M_{ix} M_{iy} M_{iz}]^T
\]

is the internal wrench by reaction forces,

\[
\{W_e\} = [F_{ex} F_{ey} F_{ez} M_{ex} M_{ey} M_{ez}]^T
\]

is the external wrench, and

\([K]\) is the fixture stiffness matrix, which is detailed in following sections.

2.1 Derivation of the Fixture Stiffness Matrix

The derivation of the fixture stiffness matrix (FSM) is discussed in detail in this section. First, the three types of coordinate system used in this paper are introduced, followed by the concept of contact point stiffness. Then, the derivation of the FSM is listed in five steps.

2.1.1 Three Coordinate Systems

There are three types of coordinate system (CS) used in this study (Fig. 2):

Global Coordinate System (GCS) – the fixed CS in 3D space. It serves as the ultimate reference frame for all other coordinate systems.

Workpiece Coordinate System (WCS) – the CS attached to each workpiece. In CAD packages, it is determined by the user at the workpiece model creation.

Local Coordinate System (LCS) – the CS attached to each contact point. It is generated based on locating position and locator orientation.

2.1.2 Contact Point Stiffness

The contact (locating/clamping) point is modelled as a linear elastic element with its stiffness in three directions, \([k_x \ k_y \ k_z]\), and in touch with workpiece surface (Fig. 3).

A contact point represents either a locating point or a clamping point, based on the circumstance. When external forces applied, the workpiece is displaced, and the contact point is displaced with the surface. The displacement of the contact point indicates the displacement of the locator/clamp. The reaction force applied on the workpiece in LCS, \(\{f^L\}\), is therefore:

\[
\begin{align*}
\{f^L_{ix}\} &= \begin{bmatrix} k_x & 0 & 0 \end{bmatrix} \cdot \left[ \begin{bmatrix} \Delta d^C_x \end{bmatrix} \right] \\
\{f^L_{iy}\} &= \begin{bmatrix} 0 & k_y & 0 \end{bmatrix} \cdot \left[ \begin{bmatrix} \Delta d^C_y \end{bmatrix} \right] \\
\{f^L_{iz}\} &= \begin{bmatrix} 0 & 0 & k_z \end{bmatrix} \cdot \left[ \begin{bmatrix} \Delta d^C_z \end{bmatrix} \right]
\end{align*}
\tag{3}
\]

or

\[
\{f^L\} = -[K] \cdot \{\Delta d^C\} \tag{4}
\]

Locator stiffness is estimated off-line using the FEA method. This stiffness can then be converted into the equivalent locating point stiffness [13].

Fig. 2. Global, workpiece, and local coordinate systems.

Fig. 3. Contact point stiffness.