Zhang Wu · Ming Xie · Qingchuan Liu · Yu Zhang

SXC control chart

Original Article

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Abstract This article proposes the SXC (sum of \( x \)s with curtailment) control chart. This chart is nearly as simple as the \( \bar{X} \) chart for the operators to understand and implement. Meanwhile, it is as effective as the CUSUM chart for detecting the process mean shifts. The SXC chart can be applied to random inspection, as well as its special cases, i.e. uniform and 100% inspections. It is found that the SXC chart can reduce the out-of-control average time to signal (ATS) by 74\%, on average, over a wide range of mean shifts compared to the Shewhart \( \bar{X} \) chart.

Keywords Control chart · Curtailment · Quality control · Statistical process control

1 Introduction

In statistical process control (SPC) practice, when dealing with a quality characteristic \( y \) that is a variable, it is usually necessary to monitor both the mean value of \( y \) and its variability [1]. In this study, in order to focus on the objective of the control charts with curtailment, only the \( \bar{X} \) control chart for monitoring mean shift is considered (the curtailment can also be incorporated into other charts, such as the \( R \) chart for monitoring process variance). The Shewhart \( \bar{X} \) control chart has been used widely in industry due to its simplicity in understanding and operation. However, the effectiveness of the Shewhart \( \bar{X} \) chart is not quite satisfactory, especially when the magnitude of the mean shift is small or moderate. In order to mitigate this problem, several supplemental runs rules are recommended to be incorporated with the \( \bar{X} \) chart in the Western Electric handbook [2]. Keats and Miskulin [3] optimized the charting parameters (e.g. sample size \( n \)) of the \( \bar{X} \) chart in order to minimize the ATS for a specified mean shift. Wu and Speeding [4] proposed a synthetic control chart which integrates the Shewhart \( \bar{X} \) chart with the conforming run length (CRL) chart.

Other researchers have developed the more sophisticated CUSUM [5] and EWMA [6] control charts. While both of these charts excel the Shewhart \( \bar{X} \) chart in terms of detection effectiveness, they are more difficult for the operators to use and understand [7]. Crowder et al. [8] mentioned that some people even believe that the CUSUM and EWMA charts are impossibly complicated and can never be taught to operators who are just smart enough to maintain and read the Shewhart \( \bar{X} \) chart. As a result, these two charts do not seem to have achieved widespread applications beyond the chemical process industry [9]. To run a CUSUM chart, two statistics, \( C^+ \) (for increasing mean shift) and \( C^- \) (for decreasing mean shift), need to be updated and plotted for every sample.

\[
\begin{align*}
C^+_0 &= 0 \\
C^+_t &= \max (0, C^+_{t-1} + \bar{Y}_t - \mu_0 - k) \\
C^-_0 &= 0 \\
C^-_t &= \min (0, C^-_{t-1} + \bar{Y}_t - \mu_0 + k),
\end{align*}
\]

where \( \mu_0 \) is the in-control value of the process mean (the target value of the quality statistic \( y \)), \( \bar{Y}_t \) is the sample mean of the \( t \)th sample, and \( k \) is the reference parameter.

The SXC chart proposed in this article also monitors the process mean shifts. It aims at achieving higher effectiveness compared to the Shewhart \( \bar{X} \) chart and, in the meantime, retaining, to a large extent, the simplicity of the Shewhart \( \bar{X} \) chart in operation and understanding.

The curtail technique is embedded in the SXC chart to enhance effectiveness. Curtailment is widely used in acceptance sampling plans in which the inspection of a
sample is terminated and the associated lot is rejected as soon as the number of observed nonconforming units exceeds the acceptance number \(1\). The operation of an SXC chart is similar. To run an SXC chart, \(n\) units in a sample (\(n\) is the sample size) are to be inspected one by one. For each unit, the deviation \(x\) of the quality characteristic \(y\) from the target \(\mu_0\) is calculated.

\[
x = y - \mu_0
\]  
(3)

Any moment, if the sum of the \(x\) values of the first \(m\) \((1 \leq m \leq n)\) inspected units is smaller than the lower control limit \(LCL\) of the SXC chart or larger than the upper control limit \(UCL\), the inspection is terminated immediately and the process is signalled as out of control. If this does not happen up to the end of the entire sample, the process is thought in control.

It is clear that curtail mechanism may signal the out-of-control condition before the completion of the inspection of the entire sample and, therefore, the signalling speed could be expedited. In fact, curtailment is the distinctive feature of the SXC chart compared to the conventional \(X\) chart.

The effectiveness of the control chart is usually measured by the average time to signal (ATS) which is defined as the average time required to signal an out-of-control condition after its occurrence. The smaller the is the out-of-control ATS, the earlier the problem is signalled and the more effective is the control chart. A small value of ATS immediately leads to the reduction of the defectives and directly benefits the industry.

For the convenience of description, the SXC chart is explained mainly for 100% inspection. Under 100% inspection, there may be no natural divisions for the products into groups of units that would form the samples for the SXC chart. Many a time, in a production line, the number of units produced in a working shift may be much greater than the sample size (say, 4 to 6, as used by the Shewhart \(X\) chart), the notation of rational subgrouping (based on the working shifts) is not enforced. The samples could be formed by designating \(n\) consecutive units as a sample for administrative convenience or just artificially \([1, 10, 11]\). In fact, Hawkins and Olwell \([12]\) argued that “the idea of rational grouping plays no useful part in CUSUM design and use.”

Under 100% inspection, the sampling interval \(h\) is equal to the product of the sample size \(n\) and the time \(t\) required to produce a unit. Thus the ATS of a control chart can be calculated as follows:

\[
ATS = h \cdot ARL = n \cdot t \cdot ARL, 
\]  
(4)

where \(ARL\) is the average run length, i.e. the average number of samples required to signal the out-of-control condition. In this study, interval \(t\) is always used as the time unit (or in other words, \(ATS\) is measured in terms of \(t\)). Therefore, for the 100% inspection,

\[
ATS = n \cdot ARL. 
\]  
(5)

When applying the control chart to random inspection where one out of \(R\) units from the production line is inspected, the sampling interval \(h\) is equal to \(\mu_R \cdot n\) (where \(\mu_R\) is the mean value of the random number \(R\)). Correspondingly,

\[
ATS = h \cdot ARL = \mu_R \cdot n \cdot ARL. 
\]  
(6)

A special case of the random inspection is the uniform inspection in which \(R\) is a constant. So,

\[
ATS = h \cdot ARL = R \cdot n \cdot ARL. 
\]  
(7)

Obviously, the 100% inspection is just a special case of the uniform inspection (where \(R=1\)).

If the quality characteristic \(y\) (in Eq. (3)) follows a normal distribution, the deviation \(x\) is also normally distributed and has a zero mean and a same standard deviation \(\sigma\) as \(y\). The central line CL of the SXC chart is therefore equal to zero. When the process is out of control due to a mean shift \(\delta\sigma\), then, \(x \sim N(\delta\sigma, \sigma^2)\).

2 Design of the SXC chart

For the design of an SXC control chart, the following parameters need to be specified:

- \(\sigma\) The process standard deviation.
- \(\delta_d\) The specified mean shift in terms of \(\sigma\) used for the design.
- \(\tau\) The specified in-control Average Time to Signal \(ATS_0\).

The value of \(\sigma\) is usually estimated from the data observed in the pilot runs. The specified \(\delta_d\) is a critical mean shift that should be detected as quickly as possible. The value of \(\tau\) is determined with regard to the false alarm rate.

In this section, the formulae for calculating the ATS of the SXC chart is first derived. Then, the optimization design of the SXC chart is presented. In this section and the next (performance comparison), the formulae are developed based on 100% inspection.

2.1 Calculation of ATS

There are two key samples: the shifting sample and the signalling sample. The shifting sample is the one in which the mean shift takes place and the signalling sample is the one in which the mean shift is detected. The signalling sample may be the same as the shifting sample or any subsequent sample.

Let \(ats(i)\) be the ATS value when the signalling sample is the \(i\)th sample counted from the shifting sample (i.e. the shifting sample is the first sample) and \(P(i)\) be the prob-