The risk-return relation and VIX: evidence from the S&P 500

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Abstract A significantly positive risk-return relation for the S&P 500 market index is detected if the squared implied volatility index (VIX) is allowed for as an exogenous variable in the conditional variance equation of the parsimonious GARCH(1,1) model. This result holds for both daily and weekly observations, for extended conditional mean and variance specifications, and is robust to sub-samples. We show that the conditional variance obtained from the GARCH model with VIX has better predictive ability for realized volatility than the conditional variance from GARCH without VIX and VIX itself, thereby documenting an important information content of VIX for conditional variance. The results are interpreted as evidence that adding VIX squared in the conditional variance equation yields a better measure of conditional variance which, subsequently, uncovers a strong risk-return relation.

Keywords S&P 500 · VIX · GARCH-M · Risk-return relation · Predictive ability

JEL Classification G12 · C22

1 Introduction

Merton (1973) Intertemporal Capital Asset Pricing Model (ICAPM) implies that the conditional expected excess market returns and the conditional variance of the market are positively related:

\[ E_t(r_{t+1}) = \mu + \lambda \text{Var}_t(r_{t+1}), \] (1)
where \( \lambda \) is the parameter measuring the degree of relative risk aversion of the representative agent. Although this relation is fundamental in finance, previous empirical analyses yielded conflicting results as to whether this relation is strong or weak, with several studies concluding that it can even be negative.\(^1\) The difficulty in detecting a strong and positive relation is attributed to the fact that the conditional variance is unobservable (Ghysels et al. 2005, p. 510). Early contributions, in elaborating the issue of non-observability of conditional variance, introduced various asymmetric GARCH-in-mean (GARCH-M) specifications, such as the exponential GARCH (EGARCH) by Nelson (1991), the threshold ARCH (TARCH) by Rabemananjara and Zakoian (1993), and the asymmetric GARCH (GJR-GARCH) by Glosten et al. (1993), and found mainly a negative relation.\(^2\) More recently, Li et al. (2005) employed a flexible semi-parametric specification of conditional variance and found evidence of a significantly negative relation for 6 markets including the USA, and Guo and Neely (2008) employed an Asymmetric Component GARCH-M (A-CGARCH-M) specification and found some evidence of a positive relation.\(^3\)

A strand of literature has focused on the role of options-implied volatility in assessing the risk-return relation. Day and Lewis (1992), adopting GARCH-M and EGARCH-M models and weekly data, explored the information content of implied volatility by adding the implied volatility in the conditional variance equation as an exogenous variable. Focusing on the period November 1983–December 1989 and assuming conditional normality in the estimation of GARCH models, they found that adding the implied volatility does not uncover a positive and significant risk-return relation (Table 2, p. 277; Table 3, p. 280). They also reached the conclusion that neither implied volatilities nor conditional volatilities from GARCH models completely characterize within-sample conditional stock market volatility. Christensen and Prabhala (1998), using monthly data and covering the period November 1983–May 1995, analyzed the information content of implied volatility. Using a regression-based research design, they found that implied volatility does predict future realized volatility in isolation as well as in conjunction with the history of past realized volatility. These results are robust to variations in econometric approach. Bollerslev and Zhou (2006) adopted monthly data for the S&P 500 index spanning the period January 1990–February 2002 and, using regression-based econometric approaches, found a positive albeit insignificant risk-return relation. Bali and Peng (2006) employed daily data for the S&P 500 and the CRSP value-weighted index for 1986–2002 and 1982–2002, respectively, and,\(^1\)


\(^2\) Several important alternatives to the GARCH-M framework have appeared in the literature. Pagan and Hong (1991) used non-parametric techniques and found a negative risk-return tradeoff, and Campbell (1987), Harvey (1991) and Whitelaw (1994) used an instrumental variables specification and obtained mixed results. Recently, Ghysels et al. (2005) employed a mixed data sampling (MIDAS) approach, and detected a positive relation. Chung et al. (2008) has shown that implied volatility has more powerful explanatory ability than MIDAS.\(^3\)

\(^3\) Conrad and Mammen (2006), using both parametric and non-parametric methods, concluded that there is no empirical evidence against the parametric specification reflected in the GARCH-M framework.