The multichoice consistent value*

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Abstract. We consider multichoice NTU games, i.e., cooperative NTU games in which players can participate in the game with several levels of activity. For these games, we define and characterize axiomatically the multichoice consistent value, which is a generalization of the consistent NTU value for NTU games and of the multichoice value for multichoice TU games. Moreover, we show that this value coincides with the consistent NTU value of a replicated NTU game and we provide a probabilistic interpretation.

Key words: NTU games, consistent NTU value, multichoice value

1. Introduction

Maschler and Owen (1989, 1992) introduced a generalization of the Shapley value for non-transferable utility (NTU) games: the consistent NTU value (or consistent Shapley value). In the first paper, they defined the consistent NTU value for hyperplane games. In the second paper, they extended this value to the general class of NTU games. Hart and Mas-Colell (1996) characterized it and proved that this value is also the solution of an n-person bargaining problem.

On the other hand, Hsiao and Raghavan (1992, 1993) extended cooperative TU games in characteristic function form to multichoice games. They

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proposed an extension of the Shapley value for these games. The idea is that in the classical formulation of a TU game, each player in any coalition has only two options: either quit the coalition or stay in it and put himself in the highest levels of cooperation to maximize the total utility of the coalition. They modelled games with many possible actions for players and gave some examples to justify their model. Nouweland et al. (1995) proposed a different extension of the Shapley value for multichoice TU games, called the multichoice value. In the setting of cost allocation problems Moulin (1995) gives also an extension of the Shapley value which is called the discrete Aumann-Shapley pricing method. In Calvo and Santos (1997) it is shown that both extensions are in fact the same solution.

In this paper, we extend the multichoice value for TU games to multichoice NTU games. The extension that we propose also extends the consistent NTU value. In order to motivate this work, let us consider the following example.

Example 1. Let A and B be two players and let V be the following NTU game:

\[ V(A) = V(B) = \{ x : x \leq 2 \} \quad \text{and} \quad V(A, B) = \{ (x, y) : x + 2y \leq 30 \} \]

The consistent value of this game can be calculated as follows. For each order of the players, the marginal contributions of the players are:

order AB:
\[
\begin{align*}
d_A &= \max \{ x : x \in V(A) \} = 2 \\
d_B &= \max \{ y : (d_A, y) \in V(A, B) \} = 14
\end{align*}
\]

order BA:
\[
\begin{align*}
d_B &= \max \{ x : x \in V(B) \} = 2 \\
d_A &= \max \{ x : (x, d_B) \in V(A, B) \} = 26
\end{align*}
\]

The consistent value is the vector of expected marginal contributions where each one of the orders is equally likely. That is, \( \varphi_A(V) = 14 \) and \( \varphi_B(V) = 8 \).

Now, suppose that player B can participate in the game with another effort level (for example, more time or more capital) and the new NTU game is:

\[ V'(A) = \{ x : x \leq 2 \}, \quad V'(B) = \{ x : x \leq 10 \} \quad \text{and} \quad V'(A, B) = \{ (x, y) : x + y \leq 30 \} \]

The consistent value of this game is \( \varphi_A(V') = 11 \) and \( \varphi_B(V') = 19 \).

The question which arises now is, which is the appropriate solution for this situation? Player A is not interested in player B employing more effort and prefers to play game \( V \), but player B prefers game \( V' \). We propose the following solution: consider a new game \( W \); two players A and B, and different actions for these players:

Player A has two actions: participate, \( \sigma_1 \), or not participate, \( \sigma_0 \).

Player B has three actions: participate as in \( V \), \( \eta_1 \), as in \( V' \), \( \eta_2 \), or not participate, \( \eta_0 \).

The multichoice NTU game \( W \) is defined on the actions of the players, that is: