Strategy-proof and efficient allocation of an indivisible good on finitely restricted preference domains*

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Abstract. We consider allocation mechanisms in economies with a single indivisible good and money. First, we show that there is no strategy-proof and Pareto efficient mechanism on some preference domains which consist of a sufficiently large but finite number of quasi-linear preferences. Second, we show that there is no strategy-proof, Pareto efficient, and equally compensatory mechanism on arbitrary preference domains which consist of more than three quasi-linear preferences.

Key words: Strategy-proofness, efficiency, domain restriction

1. Introduction

We consider economies with a single indivisible good and a transferable good. The indivisible good can be consumed by only one agent. The transferable good, regarded as money, is used for compensation. We consider allocation mechanisms which determine who consumes the indivisible good and how much compensation the other agents receive on the basis of preferences of agents. We regard the following axioms as desiderata for mechanisms. The first axiom is strategy-proofness. A mechanism is strategy-proof if truthful revelation of preferences is a dominant strategy. The second one is Pareto

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efficiency. A mechanism is Pareto efficient if it always chooses a Pareto efficient allocation. We study the possibility of designing strategy-proof and Pareto efficient mechanisms.

The possibility of designing strategy-proof mechanisms depends on the size of the preference domain of the mechanisms. In a social choice framework, Gibbard (1973) and Satterthwaite (1975) establish the impossibility of strategy-proof mechanisms when the preference domain is “unrestricted”, whereas Moulin (1980) and Barberà and Jackson (1994) characterize a rich class of strategy-proof, Pareto efficient, and anonymous mechanisms when the preference domain is restricted to “single peaked” preferences.

In two-agent pure exchange economies, Zhou (1991) shows that there is no strategy-proof, Pareto efficient, and non-dictatorial mechanism on the usual economic preference domain, and Schummer (1997) proves the same impossibility result even when the preference domain is restricted to (i) “homothetic” preferences, or (ii) more than three “linear” preferences. Therefore, the impossibility of strategy-proof and Pareto efficient mechanisms is well established in two-agent case.

However, when we consider economies with private goods, there is a crucial difference between the two-agent case and the case of more than two agents. Satterthwaite and Sonnenschein (1981) point out that there exist strategy-proof, Pareto efficient, and non-dictatorial mechanisms in the case of more than two agents. However, it is very difficult to characterize such strategy-proof mechanisms because of the concept of strategy-proofness and the presence of private goods. When some agent (e.g. agent 1) changes his preference and others remain unchanged, strategy-proofness puts constraint on agent 1’s consumption bundle directly, but on other agents’ consumption bundles indirectly (e.g. through budget balance). Satterthwaite and Sonnenschein (1981) introduce the non-bossiness condition to overcome this difficulty. Barberà and Jackson (1995) also use non-bossiness in order to characterize the set of strategy-proof and anonymous mechanisms in the case of more than two agents. However, we do not invoke non-bossiness in this paper since the economic interpretation of non-bossiness is not so clear.

We consider the possibility of strategy-proof and Pareto efficient mechanisms in economies with an indivisible good and money. A general result of Holmström (1979) implies that there is no strategy-proof and Pareto efficient mechanism on the set of all quasi-linear preferences.\(^1\) First, we consider some finite restrictions of the preference domain in order to understand how strong the impossibility result is. In Theorem 1, we show that there is no strategy-proof and Pareto efficient mechanism on a sufficiently large but finite number of quasi-linear preferences. The impossibility result holds true even on finitely restricted preference domains. A possible drawback of the above theorem is that the preference domains contain a large number of preferences when the number of agents is large. Next, we impose an additional axiom “equal compensation” and consider the possibility of such mechanisms on small preference domains. In Theorem 2, we show that there is no strategy-proof, Pareto efficient, and equally compensatory mechanism on arbitrary preference domains.

\(^1\) To escape the impossibility result, one may weaken the incentive criterion from strategy-proofness to Bayesian incentive compatibility (d’Aspremont and Gérard-Varet, 1979; Myerson and Satterthwaite, 1983).